215A Quantum Mechanics I

Course description

Graduate level introductory quantum mechanics course. Topics to be covered include mathematical framework of linear vectors spaces, basics of quantum mechanics, harmonic oscillator, coherent states, standard Schrodinger equation problems, density matrices and entanglement, Bell's inequality, uncertainty relations, path integrals and symmetry breaking, and the classical limit. Roughly corresponds to Chapters 1-9 from Shankar (2nd Ed).

Detailed syllabus

Mathematical background: Linear vector spaces, linear operators, self-adjointness and Hermiticity conditions, spectral decomposition of linear operators. (2-3 lectures)

Postulates of quantum mechanics: Basic axioms of state space, observables, Born rule. Formulation of single particle quantum mechanics, dynamics and Schrodinger equation. Position and momentum representations. Heisenberg and Schrodinger pictures for states and operators. Multi-component systems. (1-2 lectures)

Quantum harmonic oscillator: Canonical quantization in coordinate representation, Hermite polynomials. Creation-annihilation operators, Heisenberg algebra. Coherent states (2-3 lectures). Optional: general remarks on Sturm-Liouville problems

Single particle quantum mechanics: free particle and scattering states. Probability flux and unitarity. Scattering and tunneling under potential barriers. Bound sate problems: localized potential, periodic potentials (Bloch's theorem). General properties of bound state wave functions and eigenspectra. (3-4 lectures)

Non-classical features: Uncertainty principle from lack of commutativity. Density operators: formalism and computation of observables. Composite systems: entanglement, notions of separable, entangled and mixed states, von Neumann entropy. Connections to statistical mechanics: thermofield double construction. Puzzles from entanglement (EPR paradox and locality). Bell and CHSH inequalities: derivation and implication for local hidden variable models. (3 lectures)

Path integrals: The quantum propagator and the sum over paths formalism. Evaluating path integrals for quadratic systems (free particle and harmonic oscillator). Relation of path integrals to canonical quantization, connecting Euclidean time formalism to statistical mechanics. Classical limit: Ehrenfest theorem. Brief introduction to perturbation theory and Feynman diagrams. Absence of symmetry breaking and non-perturbative instanton effects (3-4 lectures)

optional: Schrodinger's equation and Hamilton-Jacobi formalism. Large quantum numbers and phase space distributions. Minimum uncertainty states. (1-2 lectures). This topic could be optional.

Symmetries: Symmetries in quantum mechanics and associated conservation laws. Discrete symmetries: parity and time-reversal (relevance to topological phases). Introduction to continuous symmetries and representation theory. (2 lectures)

Resources:

J J Sakurai, Jim Napolitano Modern Quantum Mechanics (2nd ed).

Leslie Ballantine, Quantum Mechanics: A Modern Development

Steven Weinberg, Lectures on Quantum Mechanics

R Shankar, Principles of Quantum Mechanics (2nd Ed).

P A M Dirac, Principles of Quantum Mechanics