The Northern California Physics GRE Bootcamp

Held at UC Davis, Sep 8-9, 2012

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Big tips and tricks

* Multiple passes through the exam

* Dimensional analysis (which answers make sense?)
Other hint -- look at exponentials, sines, cosines, ...

* Expansions, in particular (1+x)^n = 1 + n x + ...

* Limiting cases (e.g. make parameters go to 0 or infinity)

* Special cases (e.g. looking at circles)

* Powers of ten estimation

Big tips and tricks -- material

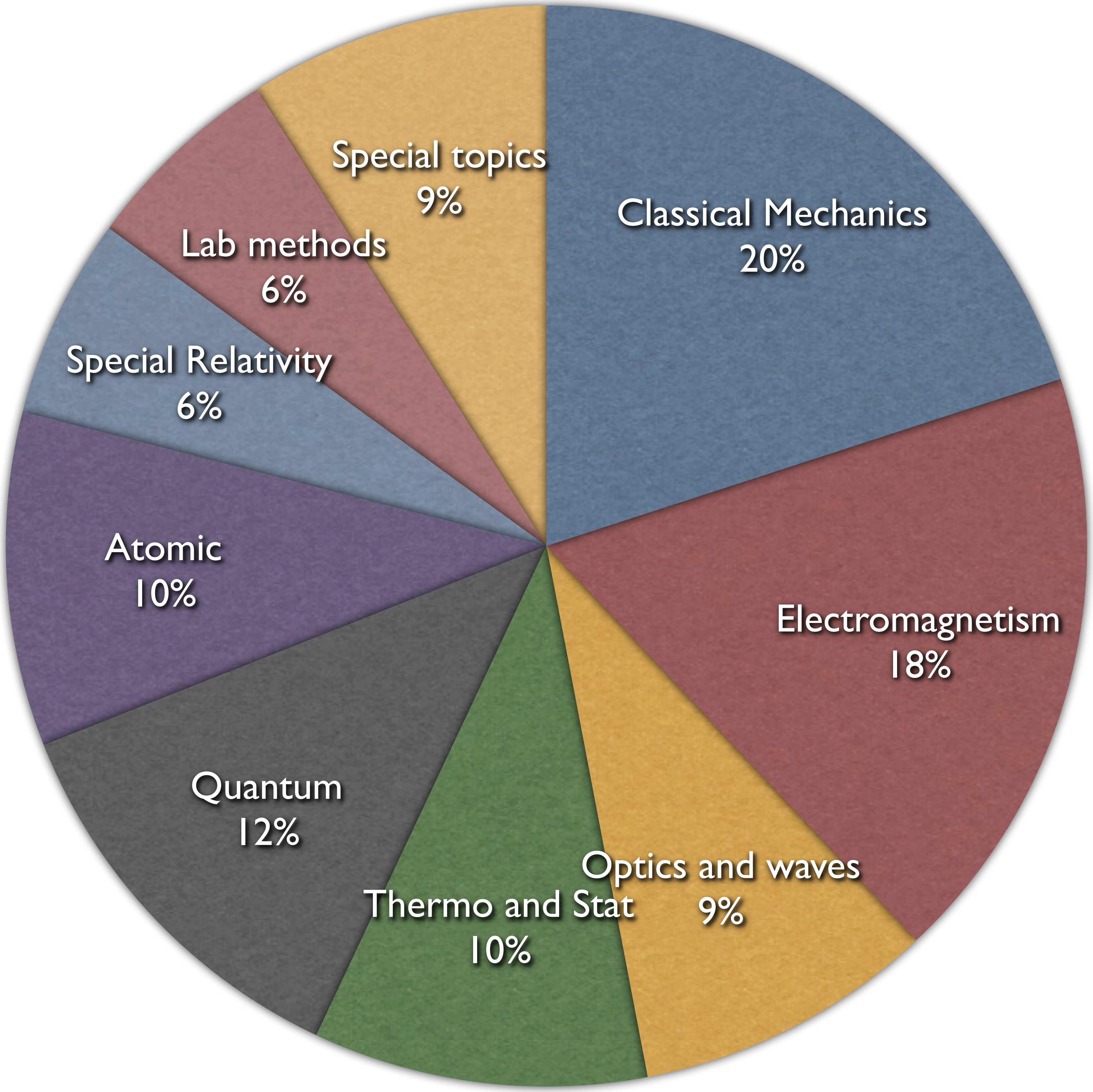
* Know your "first year" general physics really well
  - Newtonian mechanics in particular

* Worth going through
  Griffiths: Intro to electromagnetism
  Griffiths: Intro to quantum mechanics
  (Concentrate on harmonic osc, infinite square well, spin systems)
  Schroeder: Thermal physics

* Don't leave easy points on the table. You won't master everything, but if you have not had a course in something, use a std text and read the first 4 chapters.

* Look at the archive of monthly problems in *The Physics Teacher* (if you have access to a university library)

Big tips and tricks -- material



Big tips and tricks -- specifics

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Big tips and tricks -- specifics

(Very) common problems on the GRE:

- Spin 1/2 systems (triplet and singlet)
- Elastic and inelastic collisions
- Problems requiring the use of the // axis theorem
- Write down the Hamiltonian for a simple system
- Write down the Lagrangian for a simple system
- SHM

$$C = 3kN_A\left(\frac{h\nu}{kT}\right)^2 \frac{e^{h\nu/kT}}{(e^{h\nu/kT}-1)^2}$$

65. Einstein's formula for the molar heat capacity C of solids is given above. At high temperatures, C approaches which of the following?

(A) 0

(B) $3kN_A\left(\frac{h\nu}{kT}\right)$

(C) $3kN_A h\nu$

(D) $3kN_A$

(E) $N_A h\nu$

Dimensions here → $C = 3kN_A\left(\frac{hv}{kT}\right)^2 \frac{e^{hv/kT}}{(e^{hv/kT}-1)^2}$ ← This quantity is dimensionless

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(A) 0

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~~(C) $3kN_A hv$~~

(D) $3kN_A$

~~(E) $N_A hv$~~

Call $x = \frac{hv}{kT}$

$$
\begin{aligned}
C &= 3kN_A x^2 \frac{e^x}{(e^x-1)^2} \\
&= 3kN_a x^2 \left[\frac{1+x+\dots}{((1+x+\dots-1)^2}\right] \\
&= 3kN_a x^2 \left[\frac{1}{x^2}+\dots\right] \\
&= 3kN_a + \dots
\end{aligned}
$$

Suppose that a system in quantum state i has energy E_i. In thermal equilibrium, the expression

$$\frac{\sum_i E_i e^{-E_i/kT}}{\sum_i e^{-E_i/kT}}$$

represents which of the following:

1. The average energy of the system
2. The partition function
3. The probability to find the system with energy E_i
4. The entropy of the system

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This quantity is dimensionless

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represents which of the following:

1. The average energy of the system
2. ~~The partition function~~ (dimensionless)
3. ~~The probability to find the system with energy~~ $\cancel{E_i}$ (dimensionless)
4. ~~The entropy of the system~~ (J/K)

A distant galaxy is observed to have its H-beta line shifted to a wavelength of 480nm from its laboratory value of 434nm. Which is the best approximation to the velocity of the galaxy? (Note: 480/434 ~ 1.1)

a) 0.01c
b) 0.05c
c) 0.1c
d) 0.32c
e) 0.5c

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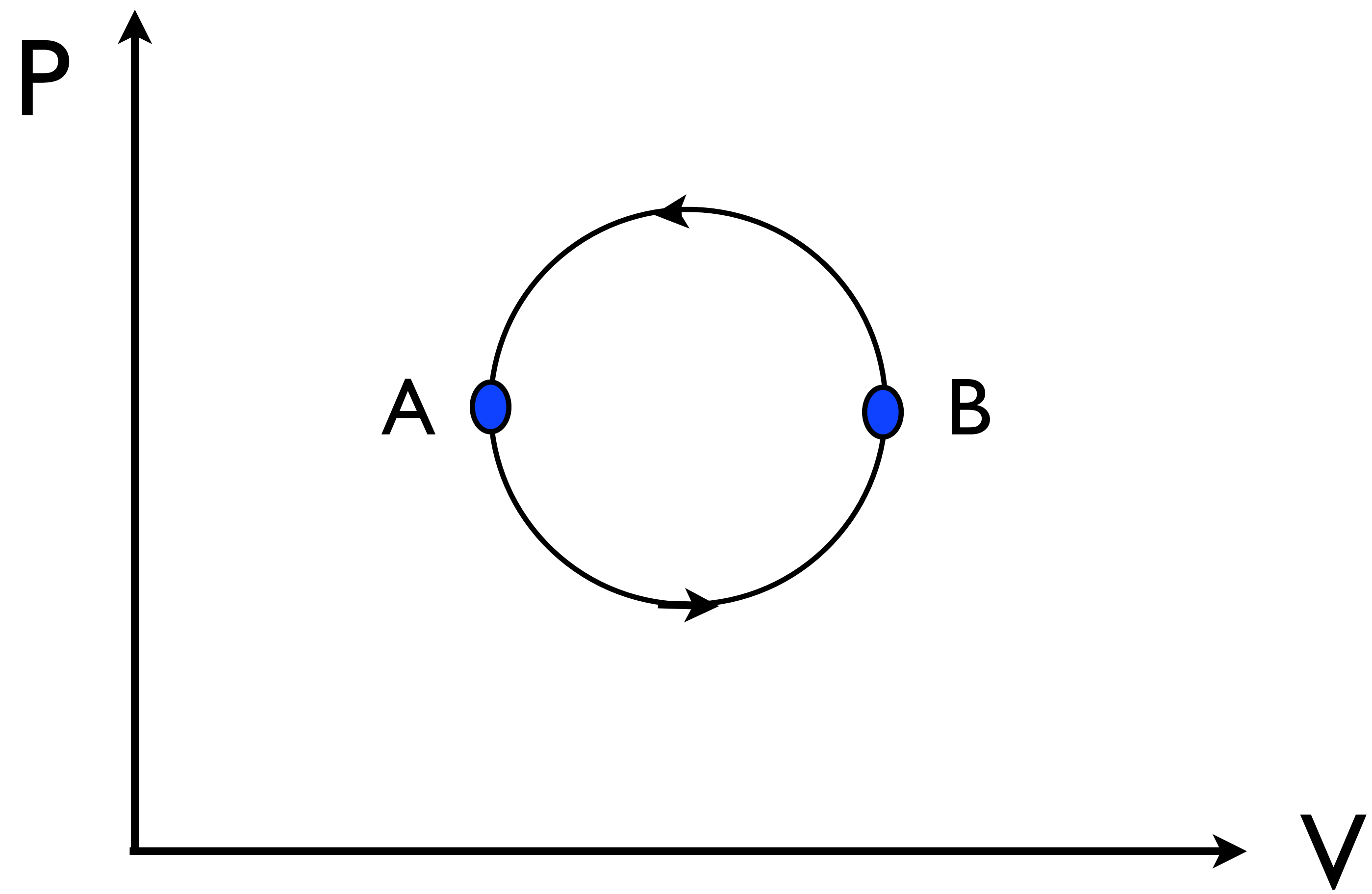
$$\lambda_{\rm obs} = \lambda_{\rm emit}\sqrt{\frac{c+v}{c-v}}$$

$$\frac{\lambda_{\rm obs}}{\lambda_{\rm emit}} = \sqrt{\frac{1+(v/c)}{1-(v/c)}} \approx \sqrt{(1+(v/c))^2}$$

$$v \approx \left(\frac{\lambda_{\rm obs}}{\lambda_{\rm emit}} - 1\right)c$$



3 moles of an ideal gas undergo a complete cycle from state A, to state B, and back to state A along the path shown

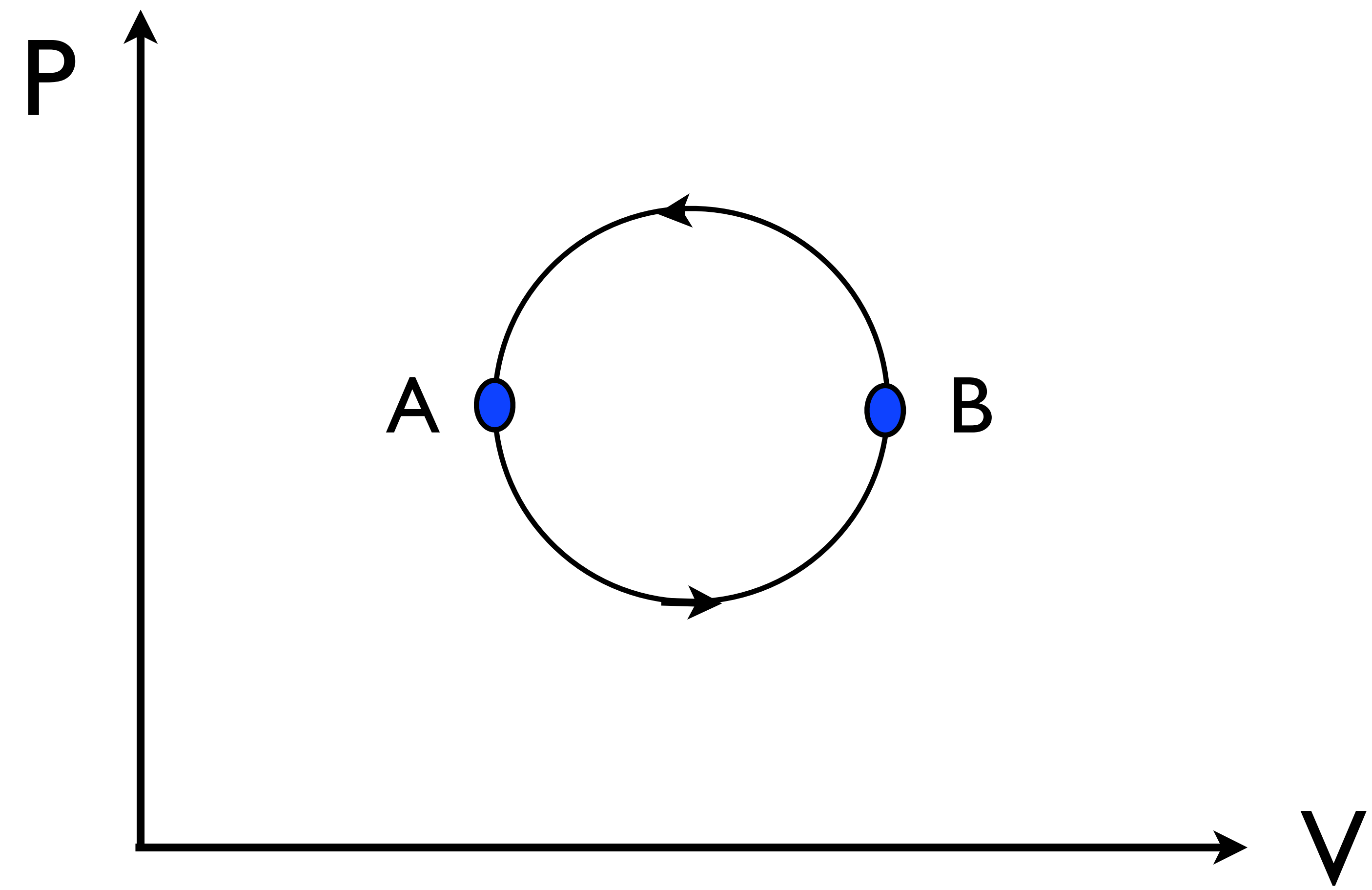
The change in (internal) **energy** over a complete cycle is:

a) positive

b) zero

c) negative

d) Not enough information



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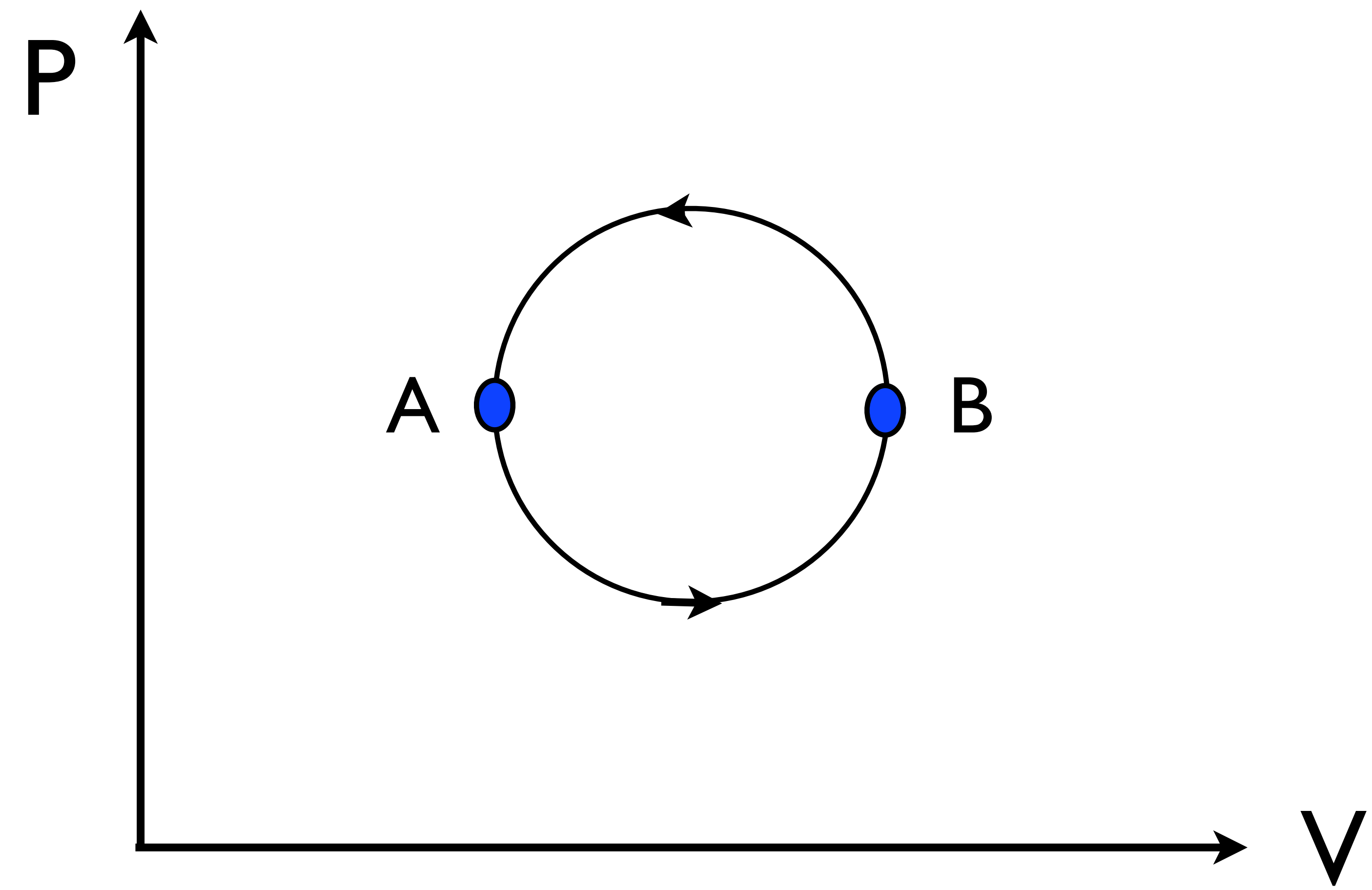
The work **done by the gas** over a complete cycle is:

a) positive

b) zero

c) negative

d) Not enough information



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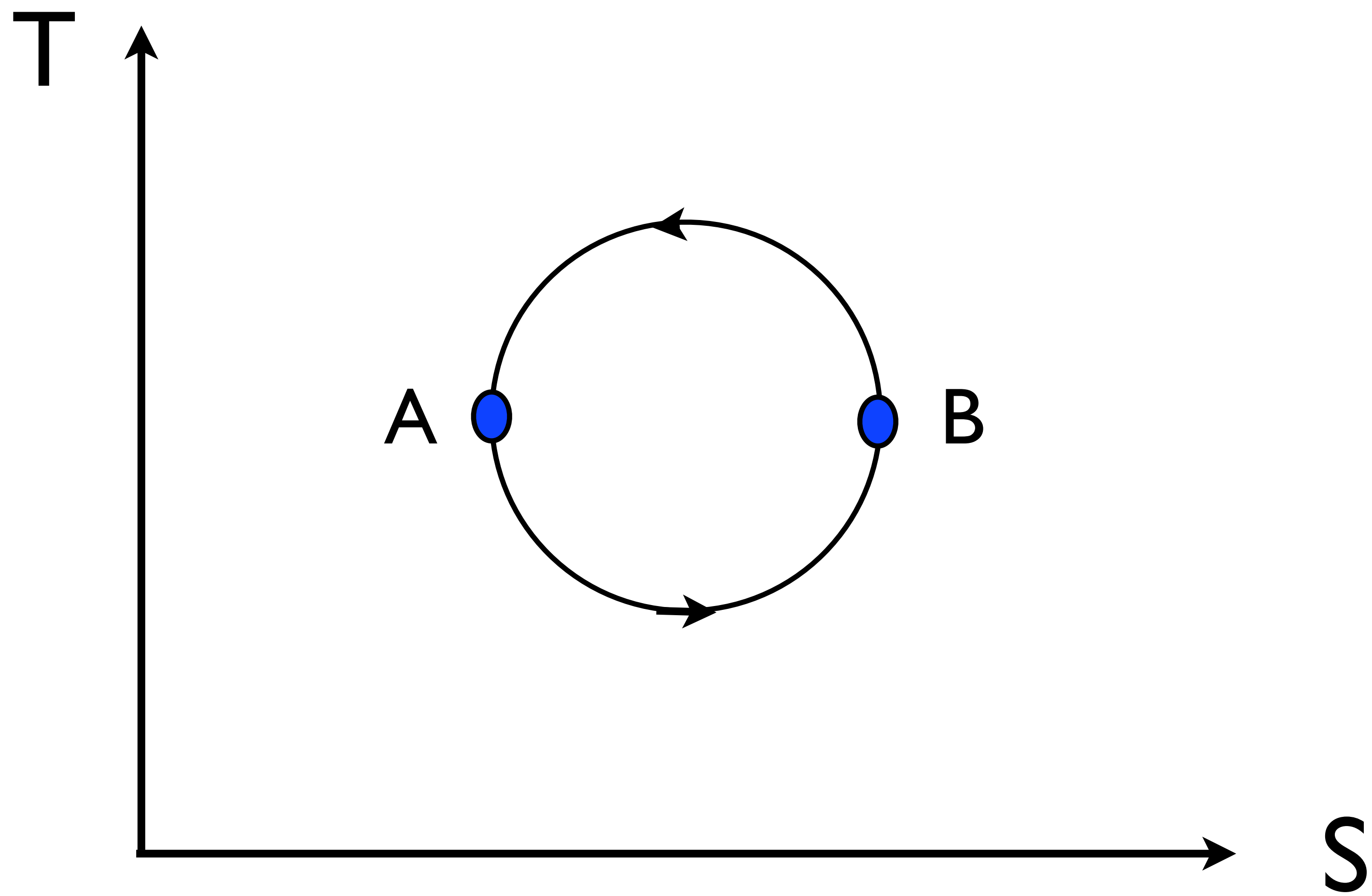
The heat absorbed by the gas over a complete cycle is:

a) positive

b) zero

c) negative

d) Not enough information



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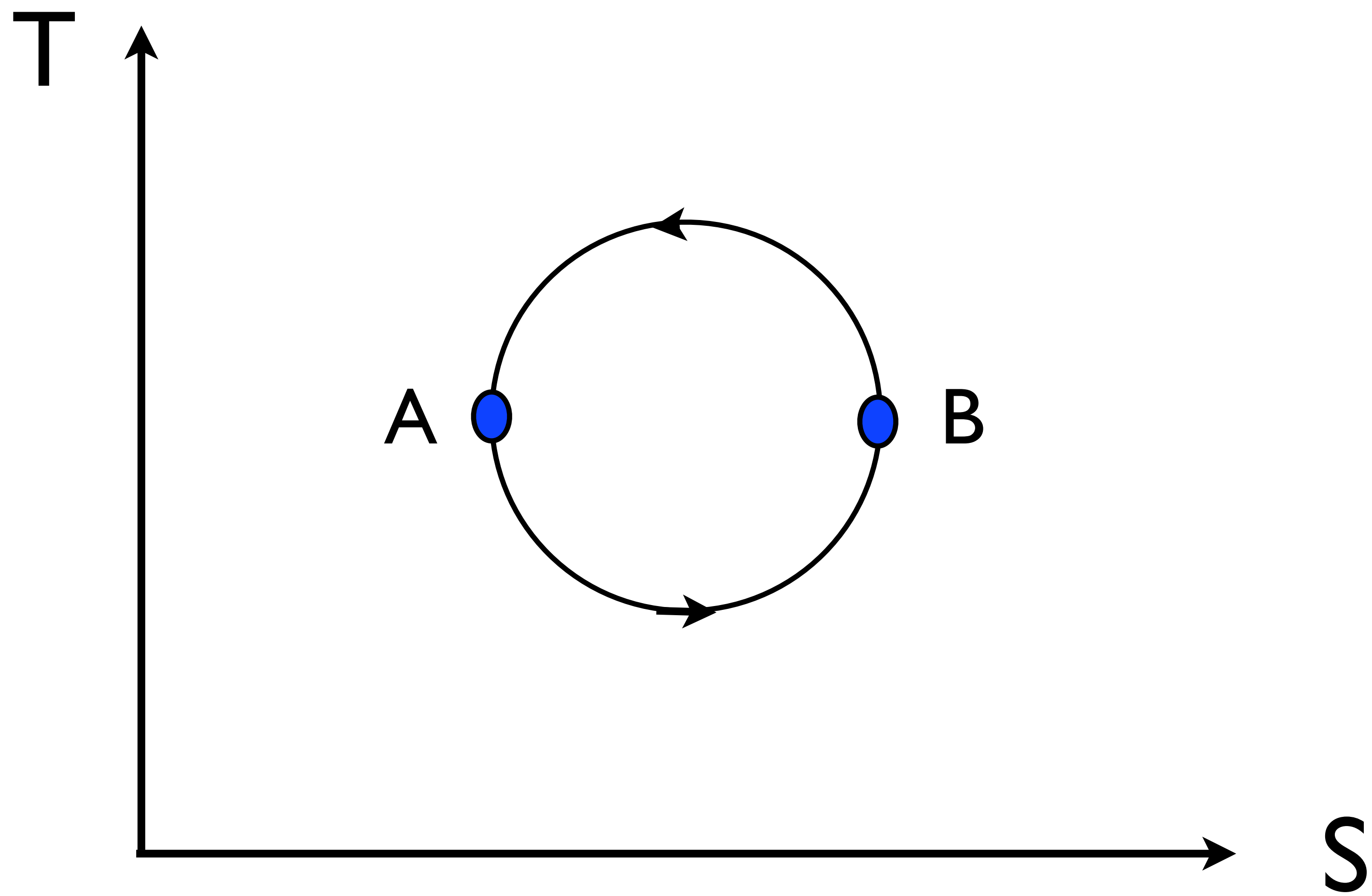
What are the signs of ΔU, the work done by the gas, and the heat absorbed by the gas?



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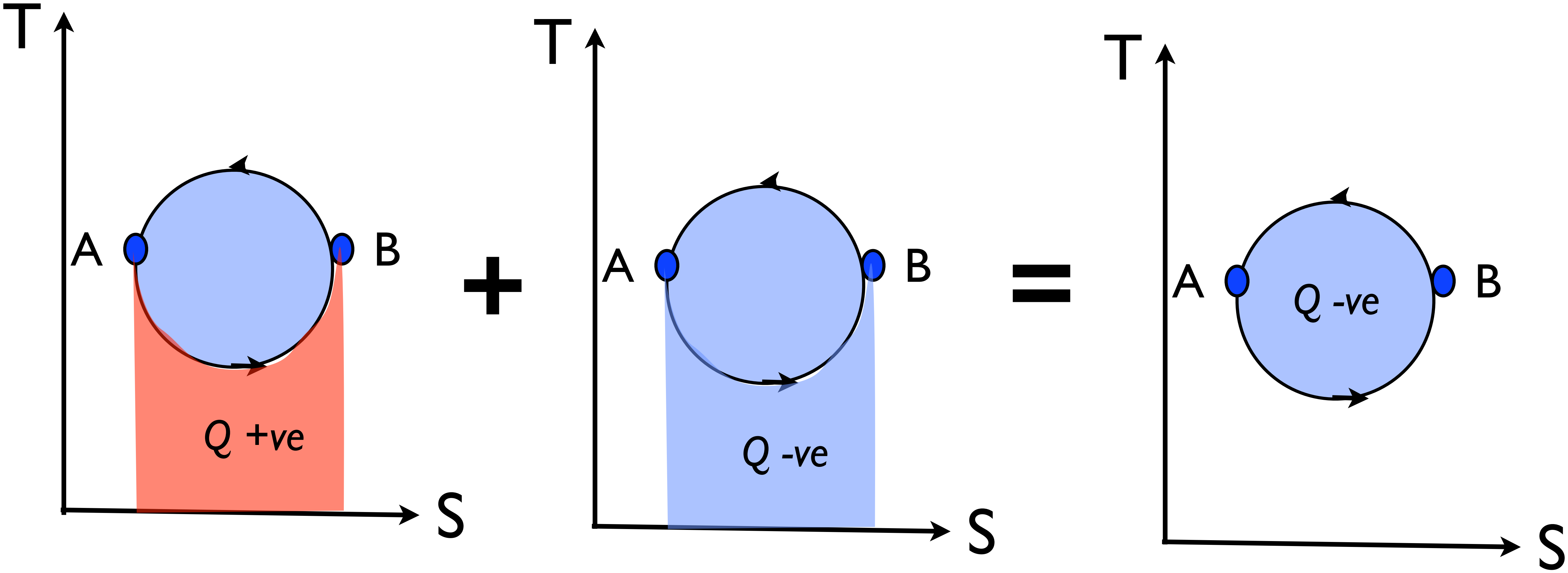
$\Delta U = 0$ (State function)



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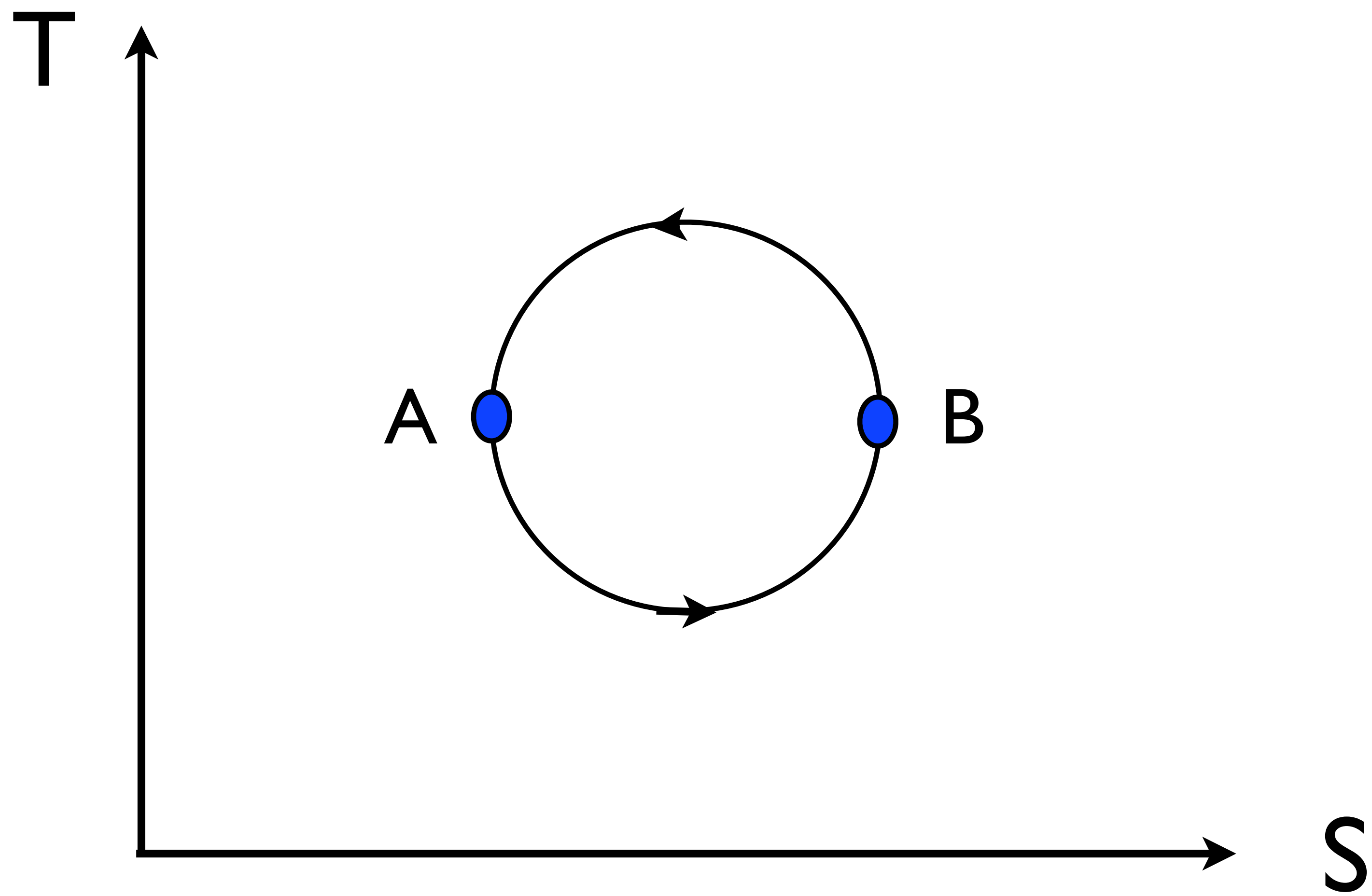
Heat absorbed = area on T-dS ($-$ve)



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Heat absorbed = area on T-dS ($-$ve)

Work done by the gas: heat absorbed is $-$ve, so work had to transfer energy *into* the gas.

So work done *by* the gas is negative (work was done on the gas)

Things to know (they always seem to come up)

1) Elastic collision formula

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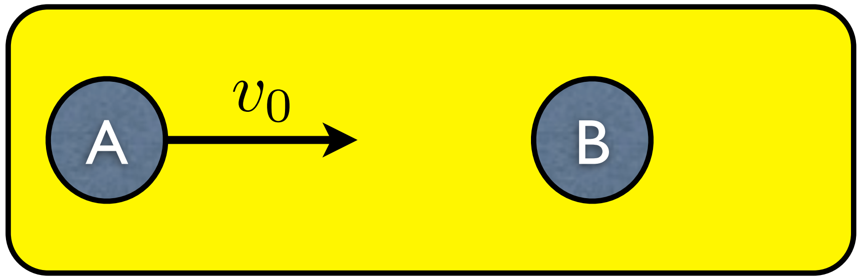
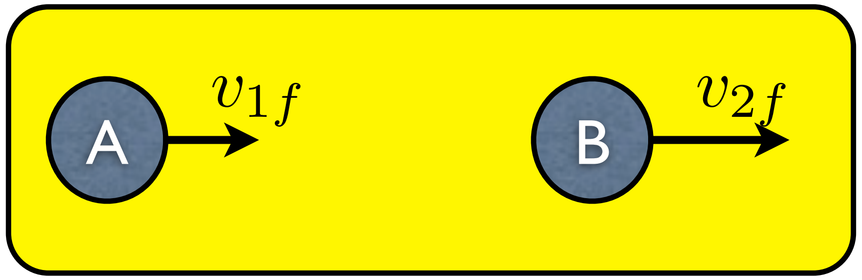


Before collision



After collision

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_0$$

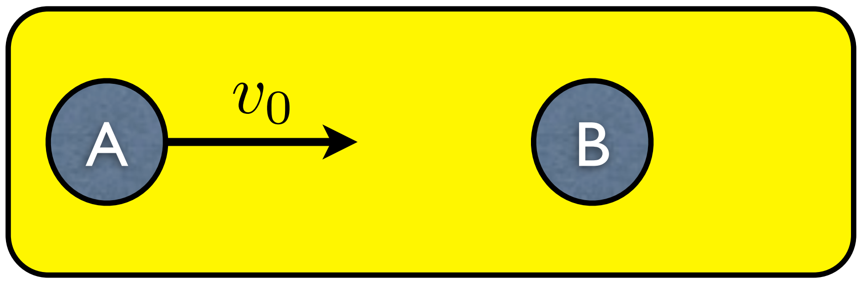
$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_0$$

$$KE = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$$

$$= \frac{p_1^2}{2m_1} + \frac{p_2^2}{2m_2}$$

Things to know (they always seem to come up)

1) Elastic collision formula



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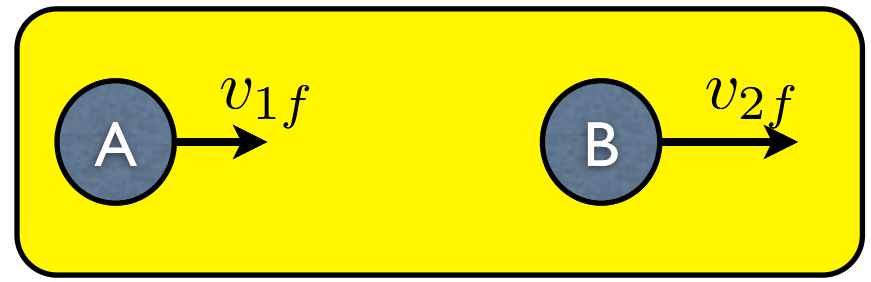

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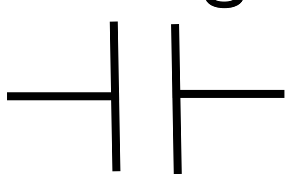
2) The limiting behavior of capacitors and inductors in DC

 acts like (while uncharged)

 (while fully charged)

(e.g. high pass filter question)

3) Virial theorem (and the quick way to get it)

m is reduced mass!

(To get levels for e.g. positronium, same formula but use reduced mass for that system)

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$$F(r) = Ar^{+n} \Rightarrow V = \frac{A}{1+n}r^{1+n} = \frac{F(r)}{1+n}r$$

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4) The Bohr formula (or know how to get it quickly)

$$\boxed{E = -\frac{Z^2(ke^2)^2 m}{2\hbar^2 n^2}}$$

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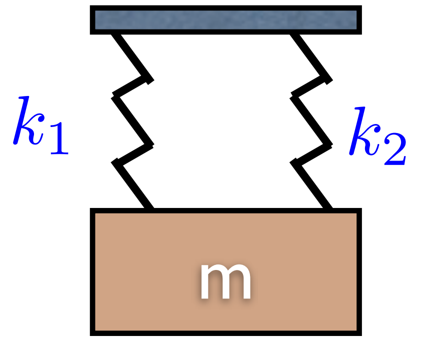
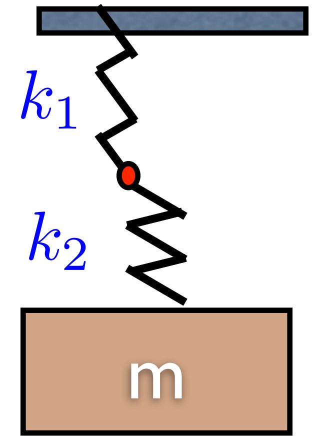
5) Combining masses, springs, capacitors, resistors





Can you find k_{equiv} ?

Frequency of oscillation?

Know reduced mass!

Which of the following is the closest to the **kinetic** energy of an electron in the ground state of Hydrogen using the Bohr model?

a) 6.8 eV
b) 13.6 eV
c) 27.2 eV
d) 0 eV
e) 40.8 eV

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Use the virial thm.

$$\langle KE \rangle + \langle V \rangle = E = -13.6\text{eV}$$

$$\langle KE \rangle - 2\langle KE \rangle = -13.6\text{eV}$$

$$\langle KE \rangle = 13.6\text{eV}$$

Which of the following is the closest to the **kinetic** energy of an electron in the ground state of Hydrogen?

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e) The kinetic energy of the ground state is not defined

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Even though E is well defined, radial position is not (see radial wavefunction). Therefore <V> is not well defined, so <KE> = E - <V> is not well defined either.

75. The period of a hypothetical Earth satellite orbiting at sea level would be 80 minutes. In terms of the Earth's radius R_e, the radius of a synchronous satellite orbit (period 24 hours) is most nearly

(A) $3\ R_e$

(B) $7\ R_e$

(C) $18\ R_e$

(D) $320\ R_e$

(E) $5800\ R_e$

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$$\frac{GM_E m_s}{R_s^2} = m_s\omega^2 R_s$$

$$\frac{GM_E}{R_s^2} = \omega^2 R_s$$

$$\frac{GM_R}{R_E^2}\frac{R_E^2}{R_s^2} = \omega^2 R_s$$

$$g\frac{R_E^2}{R_s^2} = \omega^2 R_s$$

$$R_s^3 = \frac{gR_E^2}{\omega^2}$$

mess

$$\frac{GM_E}{r^2} = \omega_1^2 r$$

$$GM_E = (2\pi)^2 r^3/T^2$$

$$\Rightarrow r^3/T^2 \text{ is constant}$$

$$r^3/(24\text{ hrs})^2 = R_e^3/(1.5\text{ hrs})^2$$

$$r = \left(\frac{24}{1.5}\right)^{2/3} R_e$$

$$= \left(\frac{24\times 2}{3}\right)^{2/3} R_e = 16^{2/3} R_e$$

$16^{1/3}$ is between 2 and 3
$16^{2/3}$ is between 4 and 9

What is the emission energy from a photon going from n = 3 to n = 1 in *positronium* (one electron and one positron orbiting one another)?

What is the emission energy from a photon going from n = 3 to n = 1 in *positronium* (one electron and one positron orbiting one another)?

Rule for Hydrogen like atoms: $E_n = \frac{-13.6 \text{ eV}}{n^2} Z^2$

But 13.6 is proportional to the *reduced* mass $m = m_{\text{electron}}$ in Hydrogen

In positronium $m = m_e/2$, so we have to halve the 13.6

$$\boxed{E_n = -\frac{6.8 \text{ eV}}{n^2}}, \qquad \text{(positronium energy levels)}$$

$$E_{\text{photon}} = E_3 - E_1 = 6.8 \text{ eV} \left(\frac{1}{1^2} - \frac{1}{3^2} \right) = 6.8 \text{ eV} \times \frac{8}{9} \approx 6 \text{ eV}$$

Situation 1) Situation 2)

Two different ways of connecting a mass *m* to two *identical springs* with spring constant k are shown above. If we denote the frequency of oscillation in situation 1 by f_1 and the frequency of oscillation in situation 2 by f_2 then f_1/ f_2 is:

a) 4
b) 2
c) 1/2
d) 1/4
e) depends on m and / or k

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(Hint: can pretend k_1 and k_2 are not the same to take limits to determine formula for k_{eff})

$$\frac{f_1}{f_2} = \frac{\omega_1}{\omega_2} = \sqrt{\frac{k_{\text{eff},1}/m}{k_{\text{eff},2}/m}} = \sqrt{\frac{k_{\text{eff},1}}{k_{\text{eff},2}}} = \sqrt{\frac{2k}{k/2}} = 2$$

A particle sits in a periodic potential

$$V(x) = d\sin(kx)$$

What is its oscillation frequency about the minimum?

A particle sits in a periodic potential

$$V(x) = d\sin(kx)$$

What is its oscillation frequency about the minimum?

Let y be the distance from the minimum. Expanding about the minimum we have:

$$V(y) = V_{\min} + \boxed{0y} + \frac{1}{2}\boxed{\frac{d^2V}{dy^2}|_{\min}}y^2 + \ldots$$

0 (because min)

Just a number, not a function

Force is

$$F = -\frac{dV}{dy} = -\frac{d^2V}{dy^2}|_{\min}y + \ldots$$

SHM with "spring constant" $k = d^2V/dy^2$ evaluated at min!

spring constant $= -dk^2\sin(kx) = +dk^2$ evaluated at min

$$f = 2\pi\sqrt{\frac{\text{spring const.}}{m}} = 2\pi\sqrt{\frac{dk^2}{m}}$$

6) Making problems look like a harmonic oscillator

$$\omega^2 = \frac{(d^2V/dx^2)|_{\rm min}}{m}$$

7) Remember spectroscopic notation (ugh)

$$^{2s+1}(\text{orbital angular momentum symbol})_j$$

and the selection rules for an electric dipole

8) Know the *pattern* of spherical harmonics

Y_ℓ^m

$$Y_0^0(\theta,\varphi) = \frac{1}{2}\sqrt{\frac{1}{\pi}} \qquad (\ell = 0)$$

$$Y_1^{-1}(\theta,\varphi) = \frac{1}{2}\sqrt{\frac{3}{2\pi}}\,\sin\theta\, e^{-i\varphi}$$

$$Y_1^{0}(\theta,\varphi) = \frac{1}{2}\sqrt{\frac{3}{\pi}}\,\cos\theta \qquad (\ell = 1)$$

$$Y_1^{1}(\theta,\varphi) = \frac{-1}{2}\sqrt{\frac{3}{2\pi}}\,\sin\theta\, e^{i\varphi}$$

$$Y_2^{-2}(\theta,\varphi) = \frac{1}{4}\sqrt{\frac{15}{2\pi}}\,\sin^2\theta\, e^{-2i\varphi}$$

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$$Y_2^{0}(\theta,\varphi) = \frac{1}{4}\sqrt{\frac{5}{\pi}}\,(3\cos^2\theta - 1) \qquad (\ell = 2)$$

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$$Y_2^{2}(\theta,\varphi) = \frac{1}{4}\sqrt{\frac{15}{2\pi}}\,\sin^2\theta\, e^{2i\varphi}$$

Too detailed ...... !

(But if you can remember these, congratulations)

8) Know the *pattern* of spherical harmonics

$$Y_\ell^m \quad \begin{array}{l} m \text{ – magnetic quantum number } (-\ell, -\ell+1, \ldots, \ell) \\ \ell \text{ – orbital quantum number } (0, 1, 2, \ldots) \end{array}$$

Y_ℓ^m contains φ dependence of the form $e^{im\phi}$

Y_ℓ^m contains ℓ dependence of the form $\sin^\ell\theta, \quad \sin^{\ell-1}\theta\cos\theta, \ldots$

(i.e. can write as ℓ sines or cosines mulitpled, or as $\sin(\ell\theta)$, $\cos(\ell\theta)$.)

Compare these rules to the spherical harmonics listed one slide ago.

We have a two dimensional infinite square well. The energy eigenstates are $|n_x, n_y\rangle$ and satisfy

$$H|n_x, n_y\rangle = (n_x^2 + n_y^2)\frac{h^2}{8mL^2}$$

Our system is in the state

$$|\psi\rangle = \frac{1}{\sqrt{14}}|1,3\rangle + \frac{3}{\sqrt{14}}|3,1\rangle - \frac{2}{\sqrt{14}}|4,1\rangle$$

What energy or energies could we get from a measurement of this state?

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a) Only $\frac{h^2}{8mL}$

b) Only 1

c) Only 0

d) Only $\frac{4h^2}{8mL}$

e) $\frac{10h^2}{8mL}$ and $\frac{17h^2}{8mL}$

f) $\frac{4h^2}{8mL}$, $\frac{10h^2}{8mL}$, and $\frac{17h^2}{8mL}$

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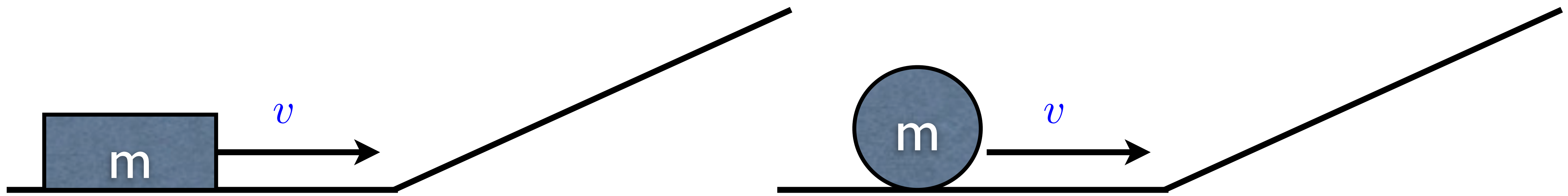
f) $\frac{4h^2}{8mL}$, $\frac{10h^2}{8mL}$, and $\frac{17h^2}{8mL}$

Random mechanics problem:

A ball and a block of mass m are moving at the same speed *v*. When they hit the ramp they both travel up it. The block slides up with (approximately) no friction, the ball experiences just enough friction to roll without slipping. Which goes higher?



a) the ball goes higher
b) the black goes higher
c) they go the same height
d) Impossible to tell from information given

Random mechanics problem:

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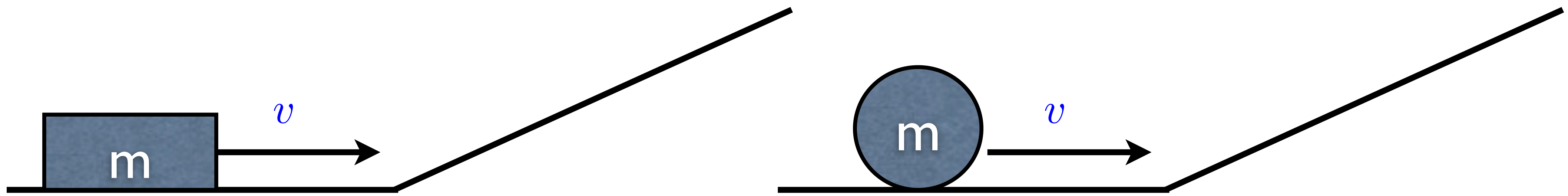

a) the ball goes higher
b) the black goes higher
c) they go the same height
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The ball has both translational kinetic energy (equal to that of the block) *and* rotational kinetic energy. Therefore

$$KE_{ball,initial} > KE_{block,initial}$$

The ball converts all this energy into potential energy, and therefore goes higher.