Quantum mechanics & Atomic physics

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GRE Boot Camp at CSU Long Beach
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Wave function probability

- $|\psi\rangle = a|1\rangle + b|2\rangle + c|3\rangle$
- $P(1) = a^2$, $P(2) = b^2$, $P(3) = c^2$, Total $P = 1$
- $<1|H|1> = E_1$, $<E> = P(1)E_1 + P(2)E_2 + P(3)E_3$
- Just do one measure, either $E_1$ or $E_2$, $E_3$.

- Statistical mechanics
- $E_1$, $E_2$, $Z = \exp(-BE_1) + \exp(-BE_2)$
- $P(E_1) = \exp(-BE_1)/Z$, $P(E_2) = \exp(-BE_2)/Z$,
- $<E> = P(1)E_1 + P(2)E_2$
- Low $T$, all on ground state.
- High $T$, equal possibility on all states.
33. A diatomic molecule is initially in the state
\[ \Psi(\Theta, \Phi) = (5Y_1^1 + 3Y_5^1 + 2Y_5^{-1})/(38)^{1/2} \], where
\[ Y_l^m \] is a spherical harmonic. If measurements are made of the total angular momentum quantum number \( \ell \) and of the azimuthal angular momentum quantum number \( m \), what is the probability of obtaining the result \( \ell = 5 \) ?

(A) \( \frac{36}{1444} \)
(B) \( \frac{9}{38} \)
(C) \( \frac{13}{38} \)
(D) \( \frac{5}{(38)^{1/2}} \)
(E) \( \frac{34}{38} \)
A system consists of $N$ weakly interacting subsystems, each with two internal quantum states with energies 0 and $\epsilon$. The internal energy for this system at absolute temperature $T$ is equal to

(A) $N\epsilon$

(B) $\frac{3}{2} NkT$

(C) $N\epsilon e^{-\epsilon/kT}$

(D) $\frac{N\epsilon}{(e^{\epsilon/kT} + 1)}$

(E) $\frac{N\epsilon}{(1 + e^{-\epsilon/kT})}$
Bohr Model

- Hydrogen like atom:
  - $E(n) = -13.6 \text{ev} \times \left(\frac{Z^2}{n^2}\right) \times \left(\frac{M\text{-reduced}}{M\text{-e}}\right)$
  - $E = E(n_f) - E(n_i)$

The energy required to remove both electrons from the helium atom in its ground state is 79.0 eV. How much energy is required to ionize helium (i.e., to remove one electron)?

(A) 24.6 eV
(B) 39.5 eV
(C) 51.8 eV
(D) 54.4 eV
(E) 65.4 eV

$13.6 \times 4 = 54.4 \text{ eV}$
$79 - 54.4 = 24.6 \text{ eV}$
Positronium is an atom formed by an electron and a positron (antielectron). It is similar to the hydrogen atom, with the positron replacing the proton. If a positronium atom makes a transition from the state with \( n = 3 \) to a state with \( n = 1 \), the energy of the photon emitted in this transition is closest to

\[
13.6 \times \frac{1}{2} \times (1 - \frac{1}{9}) = 6.8 \times \frac{8}{9} = 6\text{eV}
\]
If a singly ionized helium atom in an $n = 4$ state emits a photon of wavelength 470 nanometers, which of the following gives the approximate final energy level, $E_f$, of the atom, and the $n$ value, $n_f$, of this final state?

<table>
<thead>
<tr>
<th>$E_f$ (eV)</th>
<th>$n_f$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(A) -6.0</td>
<td>3</td>
</tr>
<tr>
<td>(B) -6.0</td>
<td>2</td>
</tr>
<tr>
<td>(C) -14</td>
<td>2</td>
</tr>
<tr>
<td>(D) -14</td>
<td>1</td>
</tr>
<tr>
<td>(E) -52</td>
<td>1</td>
</tr>
</tbody>
</table>

- $E_i = E(4) = 13.6 \times 4/16 = 3.4$ eV
- $E = h\nu/\lambda = 2.64$ eV
- $E_f = 6$ eV
- $13.6 \times 4/n^2 = 6$, $n_f = 3$
Quantum number

- \( n \) principle QN
- \( L \) orbital QN, \( L^2 \rightarrow L(L+1) \), \( L \) from 0...n-1
- \( m_L \) magnetic QN, -L.....L
- \( S \) spin, \( m_s \)
- \( J \) total angular momentum, \( L+S.....|L-S| \)
- \( M_J \)
- \( ^2S_{1/2} \) L=0 S orbit, L=1 P orbit, L=2 d orbit
- \( J=1/2, n=2, \)
- Dipole transition, change of \( L=+/- \) 1. change \( S=0 \)
A $3p$ electron is found in the $^3P_{3/2}$ energy level of a hydrogen atom. Which of the following is true about the electron in this state?

(A) It is allowed to make an electric dipole transition to the $^2S_{1/2}$ level.

(B) It is allowed to make an electric dipole transition to the $^2P_{1/2}$ level.

(C) It has quantum numbers $\ell = 3$, $j = 3/2$, $s = 1/2$.

(D) It has quantum numbers $n = 3$, $j = \ell$, $s = 3/2$.

(E) It has exactly the same energy as it would in the $^3D_{3/2}$ level.
Binding energy

• Mass difference of total mass of constituent nucleons- mass of nucleas.

The $^{238}$U nucleus has a binding energy of about 7.6 MeV per nucleon. If the nucleus were to fission into two equal fragments, each would have a kinetic energy of just over 100 MeV. From this, it can be concluded that

(A) $^{238}$U cannot fission spontaneously
(B) $^{238}$U has a large neutron excess
(C) nuclei near $A = 120$ have masses greater than half that of $^{238}$U
(D) nuclei near $A = 120$ must be bound by about 6.7 MeV/nucleon
(E) nuclei near $A = 120$ must be bound by about 8.5 MeV/nucleon

$n \times m - A = n \times E_A$
$n/2 \times m - B = n/2 \times E_B$
$A - 2 \times B = K$
$E_B - E_A = K/n$
$7.6 + 200/238 = 8.44 \text{ ev}$