GRE Physics: If you have to solve it, you're (probably) doing it wrong!

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- Because you know how to solve a problem does not necessarily mean you know how to do it on the GRE!
- For the GRE, the right answer is all that matters.
  - If you can solve a problem, but it takes you 20 minutes of hard effort, that won't help you.
  - If you can quickly work your way to the right answer you'll do much better (and this ability is what they're really testing for anyway)
- Many seemingly complicated questions are really asking one of the following:

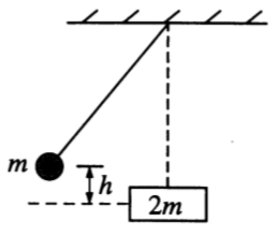
  (1) Can you reason physically?

  (2) Do you know important physics facts?
- You will **not** have time to solve everything explicitly.
  - You have just under two minutes per problem on average.
  - Many problems can be answered much faster than this.

Advice

- Practice physical reasoning (approximating, limiting cases, unit analysis, shortcuts, educated guessing)
- Know important things
  - Kepler's laws, the energy of blackbody, resonance frequency of RLC circuit, etc.
- Once you solve a practice problem, ask yourself if you could have gotten the answer faster!
- **Always** look at the available answers before diving into a difficult calculation.
  - If you can safely eliminate three answers, taking a guess isn't a bad idea. In fact, it's a good idea.
  - You will often find the answer staring you in the face, without having to solve anything.

Example 1



7. As shown above, a ball of mass m, suspended on the end of a wire, is released from height h and collides elastically, when it is at its lowest point, with a block of mass $2m$ at rest on a frictionless surface. After the collision, the ball rises to a final height equal to

(A) 1/9 h
(B) 1/8 h
(C) 1/3 h
(D) 1/2 h
(E) 2/3 h

The hard way:
$mgh = ½mv_1^2$
$mv_1 = 2mv_2 – mv_3$,
$mgh = mv_2^2 + ½mv_3^2$, etc.

It could take a while to solve this.

Easier way: Recognize that when the masses make contact, there is a reference frame in which they have zero net momentum.

- Let v_1 be the velocity of mass *m* just before the moment of impact. In a frame of reference moving at $1/3v_1$ to the right, the two masses have equal but opposite momentum (the mass *m* moves at $2/3v_1$ in this frame, and the mass 2*m* moves at $-1/3v_1$).
- In this frame they just turn around and keep the same (magnitude) velocities. So the mass *m* rebounds with velocity $-2/3v_1$, which is $-1/3v_1$ in the laboratory frame. With 1/3 it's initial strike velocity, it has lost 1/9 of its energy, and therefore will only rise to 1/9 the initial height. The answer is (A).

Example 2

16. The mean free path for the molecules of a gas is approximately given by $\frac{1}{\eta\sigma}$, where η is the number density and σ is the collision cross section. The mean free path for air molecules at room conditions is approximately

(A) 10^{-4} m
(B) 10^{-7} m
(C) 10^{-10} m
(D) 10^{-13} m
(E) 10^{-16} m

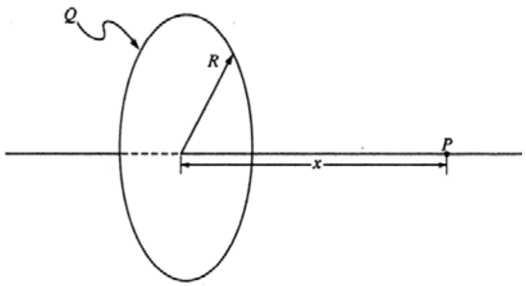
The hard way:
Try to use the information given. What is the number density of air molecules? What is their cross-section? Who knows? Well, maybe you do know, in which case this is easy.

Understand what you're looking for, which is the average distance an air molecule travels before hitting another one. Eliminate answers and take a guess.

- Answers (C), (D) and (E) are obviously incorrect. Why?

- That leaves just two plausible answers: (A) or (B). Guessing in such a case is advantageous. In this case the correct answer is (B), which you might guess because (A) is a rather large distance from the perspective of air molecules.

Example 3



4. A small particle of mass m and charge $-q$ is placed at point P and released. If $R >> x$, the particle will undergo oscillations along the axis of symmetry with an angular frequency that is equal to

(A) $\sqrt{\dfrac{qQ}{4\pi\varepsilon_0 mR^3}}$

(B) $\sqrt{\dfrac{qQx}{4\pi\varepsilon_0 mR^4}}$

(C) $\dfrac{qQ}{4\pi\varepsilon_0 mR^3}$

(D) $\dfrac{qQx}{4\pi\varepsilon_0 mR^4}$

(E) $\sqrt{\dfrac{qQx}{4\pi\varepsilon_0 m}\dfrac{1}{R^2 + x^2}}$

The hard way: Solving it (actually not that hard).

Instead you could note that the frequency must be independent of x (analogous to the spring frequency being independent of initial displacement). That rules out (B), (D) and (E). Checking the units of (A) and (C) should convince you that (A) is the answer.

You could also remember that the spring frequency is $\omega = \sqrt{\dfrac{k}{m}}$

And then look for the analogous expression among the possibilities.

Example 4

29. The characteristic distance at which quantum gravitational effects are significant, the Planck length, can be determined from a suitable combination of the physical constants G, $\hbar$, and c. Which of the following correctly gives the Planck length?

(A) $G\hbar c$

(B) $G\hbar^2 c^3$

(C) $G^2\hbar c$

(D) $G^{\frac{1}{2}}\hbar^2 c$

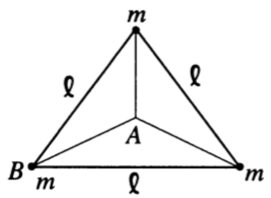
(E) $(G\hbar/c^3)^{\frac{1}{2}}$

The hard way:
In this case there really isn't a hard way. You must either know the answer, or use unit analysis / educated guessing.

The correct answer must have units of distance. Refer to the top of the exam to get the units of h-bar (J s or kg m^2/s) and G (m^3 / (kg s^2)).

The "kg" term must cancel out, so we know that the correct answer must multiply G and h-bar (or the same powers of them). This eliminates (B), (C) and (D). A quick analysis of (A) shows that it does not have units of distance. Therefore the answer is (E).

Example 5



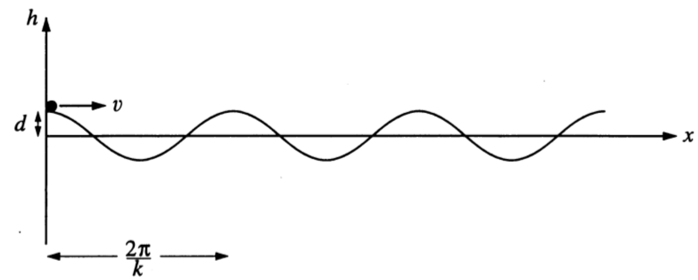
32. Three equal masses m are rigidly connected to each other by massless rods of length ℓ forming an equilateral triangle, as shown above. The assembly is to be given an angular velocity ω about an axis perpendicular to the triangle. For fixed ω, the ratio of the kinetic energy of the assembly for an axis through B compared with that for an axis through A is equal to

(A) 3
(B) 2
(C) 1
(D) 1/2
(E) 1/3

This is asking about the ratios of the moments of inertia around the two axes A and B.

- Answers (C), (D) and (E) are obviously incorrect. Why?
- A little geometry is required. You need to ratios of the moments of inertia of the two configurations. The segment from A to B is length l/sqrt(3). The moment around A is then $3m*l^2/3 = ml^2$. Around B, the moment is $2ml^2$. Therefore the answer is (B).

Example 6



83. Consider a particle moving without friction on a rippled surface, as shown above. Gravity acts down in the negative h direction. The elevation $h(x)$ of the surface is given by $h(x) = d\ \cos(kx)$. If the particle starts at $x = 0$ with a speed v in the x direction, for what values of v will the particle stay on the surface at all times?

(A) $v \leq \sqrt{gd}$

(B) $v \leq \sqrt{\frac{g}{k}}$

(C) $v \leq \sqrt{gkd^2}$

(D) $v \leq \sqrt{\frac{g}{k^2 d}}$

(E) $v > 0$

Solving it, using forces of constraint or other approaches from classical mechanics, could be very time-consuming.

This problem calls for physical reasoning, not explicit solving. Very important: Before jumping into solving a problem like this one, look at the answers!

Example 7

26. A series *RLC* circuit is used in a radio to tune to an FM station broadcasting at 103.7 MHz. The resistance in the circuit is 10 ohms and the inductance is 2.0 microhenries. What is the best estimate of the capacitance that should be used?

(A) 200 pF
(B) 50 pF
(C) 1 pF
(D) 0.2 pF
(E) 0.02 pF

Here you need to know an important result for RLC circuits. There is no other way to get this one, unless you want to waste lots of time.

- You should memorize the resonance frequency of an RLC circuit:

$$\omega = 2\pi f = \frac{1}{\sqrt{LC}}$$

- Here f is given as ~10^8 Hz. Some quick calculation will give the approximate answer.

Example 8

39. Which of the following atoms has the lowest ionization potential?

(A) $^{2}_{4}\mathrm{He}$

(B) $^{7}_{14}\mathrm{N}$

(C) $^{8}_{16}\mathrm{O}$

(D) $^{18}_{40}\mathrm{Ar}$

(E) $^{55}_{133}\mathrm{Cs}$

If you have an element further up the atomic table, it has more protons, but also more electrons. Having more electrons tends to shield the outermost electrons from the force of the nucleus. So typically it's easier to ionize elements with more electrons. Therefore the answer is (E), cesium.

Example 9

42. Light of wavelength 500 nanometers is incident on sodium, with work function 2.28 electron volts. What is the maximum kinetic energy of the ejected photoelectrons?

(A) 0.03 eV
(B) 0.2 eV
(C) 0.6 eV
(D) 1.3 eV
(E) 2.0 eV

Remember that the maximum kinetic energy of an ejected electron is the energy of the incident photon minus the work function (the energy it took to free it).

Definitely memorize the number 1240.

Any time you are told the wavelength λ of light in nanometers, the energy of the photons (in eV) is $1240/\lambda$. In this case, the photon energy is 1240/500 ~ 2.5 eV.

So the answer is just 2.5 – 2.28 ~ 0.2. (B)

Example 11

60. The Lyman alpha spectral line of hydrogen (λ = 122 nanometers) differs by 1.8×10^{-12} meter in spectra taken at opposite ends of the Sun's equator. What is the speed of a particle on the equator due to the Sun's rotation, in kilometers per second?

(A) 0.22
(B) 2.2
(C) 22
(D) 220
(E) 2200

The hard way:
Doppler effect. Not particularly difficult, if you remember the expression needed.
If you don't, and you feel the urge to derive it during the test, resist that urge.

Make an educated guess. Both (D) and (E) are very large velocities and unlikely to be the answer. (A) is probably too low, given that a point on the Earth's equator moves faster (1000 mi/hr ~ 0.5 km/s). That leaves (B) and (C). Take a guess – the answer is (B).

Example 12

18. The energy required to remove both electrons from the helium atom in its ground state is 79.0 eV. How much energy is required to ionize helium (i.e., to remove one electron) ?

(A) 24.6 eV
(B) 39.5 eV
(C) 51.8 eV
(D) 54.4 eV
(E) 65.4 eV

Two options here.

Option 1. Know the answer (Most astronomers could tell you offhand).

Option 2. Physical reasoning.

Example 13

33. If a charged pion that decays in 10^{-8} second in its own rest frame is to travel 30 meters in the laboratory before decaying, the pion's speed must be most nearly

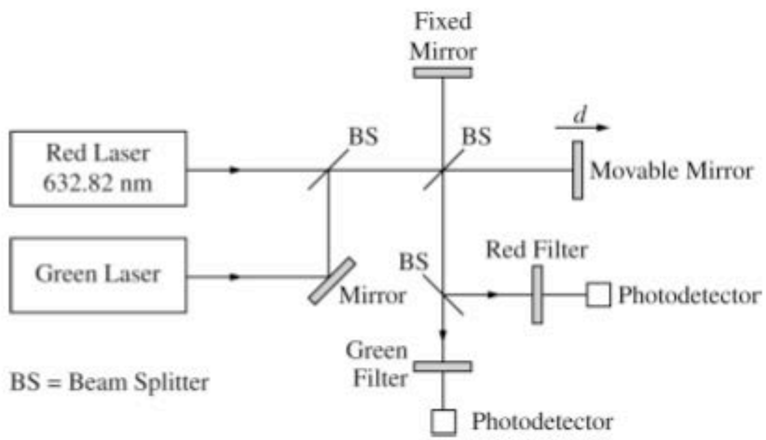
(A) 0.43×10^8 m/s
(B) 2.84×10^8 m/s
(C) 2.90×10^8 m/s
(D) 2.98×10^8 m/s
(E) 3.00×10^8 m/s

As usual, the hard way is to solve it.

There's a tricky way to get to the answer. Ask, is this a big or a small relativistic effect?

- *Without* time dilation, traveling at the speed of light, the pion could only travel $d = vt = 3\text{x}10^8 \text{ x } 10^{-8} = 3$ meters before decaying.
- We are told that with the relativistic dilation, it travels 30 meters – a BIG effect.
- Big relativistic effects only occur for velocities very close to c (>99% c). Therefore, the answer must be (D).

Example 14



100. A Michelson interferometer is configured as a wavemeter, as shown in the figure above, so that a ratio of fringe counts may be used to compare the wavelengths of two lasers with high precision. When the mirror in the right arm of the interferometer is translated through a distance d, 100,000 interference fringes pass across the detector for green light and 85,865 fringes pass across the detector for red ($\lambda = 632.82$ nanometers) light. The wavelength of the green laser light is

(A) 500.33 nm
(B) 543.37 nm
(C) 590.19 nm
(D) 736.99 nm
(E) 858.65 nm

What they are asking: What is the wavelength of green laser light?
Beware the "general knowledge" question disguised as a fiendish physics problem!

The End!

Example 10

66. A sample of radioactive nuclei of a certain element can decay only by γ-emission and β-emission. If the half-life for γ-emission is 24 minutes and that for β-emission is 36 minutes, the half-life for the sample is

(A) 30 minutes
(B) 24 minutes
(C) 20.8 minutes
(D) 14.4 minutes
(E) 6 minutes

- Don't do complicated math.
- First, if the sample can decay in two different ways, its half-life must be shorter than either individual decay half-life. That rules out (A) and (B).
- Similarly, the sample's half-life cannot be shorter than half of the shortest individual half-life. That rules out (E). Guessing is a good idea now, but one more step gets it.
- The answer must lie between half of one half-life and half of the other. So between 12 minutes and 18 minutes. The answer is (D). This actually could have been jumped to at the beginning.

3. A satellite of mass m orbits a planet of mass M in a circular orbit of radius R. The time required for one revolution is

(A) independent of M

(B) proportional to $\sqrt{m}$

(C) linear in R

(D) proportional to $R^{3/2}$

(E) proportional to R^2

This could be done by solving the orbit equations. But that would be a huge waste of time!

The way to solve this one is to know Kepler's 3rd Law:

$$T^2 \propto R^3$$

10. A 3-microfarad capacitor is connected in series with a 6-microfarad capacitor. When a 300-volt potential difference is applied across this combination, the total energy stored in the two capacitors is

(A) 0.09 J
(B) 0.18 J
(C) 0.27 J
(D) 0.41 J
(E) 0.81 J

Here you must remember the energy stored in a capacitor:

$$\frac{1}{2}CV^2$$

If you remember this, the problem is trivial – if not, it's very hard.

13. Two stars are separated by an angle of 3×10^{-5} radians. What is the diameter of the smallest telescope that can resolve the two stars using visible light ($\lambda \cong 600$ nanometers) ? (Ignore any effects due to Earth's atmosphere.)

(A) 1 mm
(B) 2.5 cm
(C) 10 cm
(D) 2.5 m
(E) 10 m

You should know that the resolving power of a telescope is:

$$\theta \sim \frac{\lambda}{D}$$

There's a numerical correction factor on this relation, but even if you don't remember it, it's okay! This basic relation will get you close enough to the right answer.

33. If a charged pion that decays in 10^{-8} second in its own rest frame is to travel 30 meters in the laboratory before decaying, the pion's speed must be most nearly

(A) 0.43×10^8 m/s
(B) 2.84×10^8 m/s
(C) 2.90×10^8 m/s
(D) 2.98×10^8 m/s
(E) 3.00×10^8 m/s

The rigorous way to solve this is to write down the time dilation factor, multiply by v, set equal to 30, and solve. But this is hard.

A much simpler method is to ask yourself, is this a big or a small relativistic effect? Note that if there were no time dilation, and the particle traveled at c, it would only get 3 meters. In fact, it gets 30 meters – a BIG effect! So the velocity must to close to (~99%) c. So D.

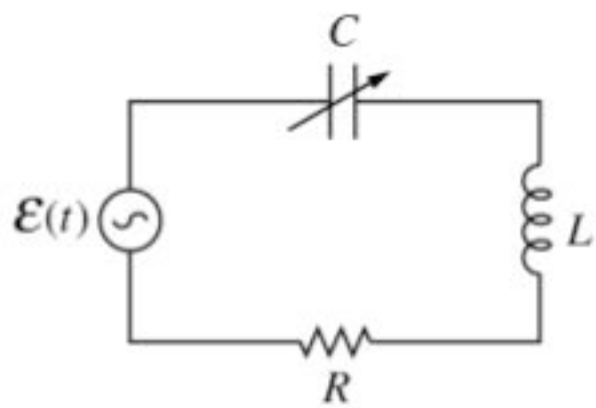
35. If the absolute temperature of a blackbody is increased by a factor of 3, the energy radiated per second per unit area does which of the following?

(A) Decreases by a factor of 81.
(B) Decreases by a factor of 9.
(C) Increases by a factor of 9.
(D) Increases by a factor of 27.
(E) Increases by a factor of 81.

You must know that the total energy emitted by a blackbody is proportional to the fourth power of its temperature.

Remember that resonance occurs when:

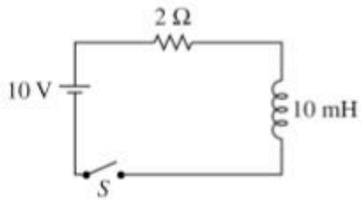
$$\omega = \frac{1}{\sqrt{LC}}$$



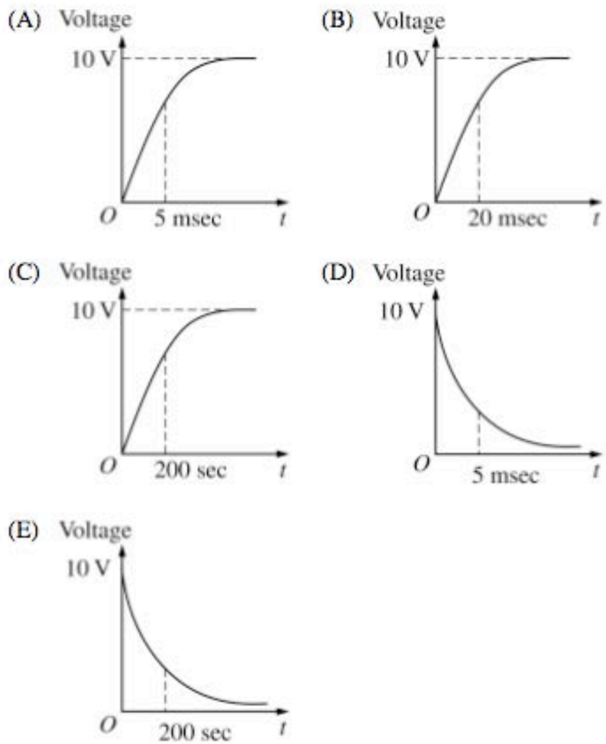
Note the extra (unnecessary) information designed to distract you…

38. An AC circuit consists of the elements shown above, with $R = 10{,}000$ ohms, $L = 25$ millihenries, and C an adjustable capacitance. The AC voltage generator supplies a signal with an amplitude of 40 volts and angular frequency of 1,000 radians per second. For what value of C is the amplitude of the current maximized?

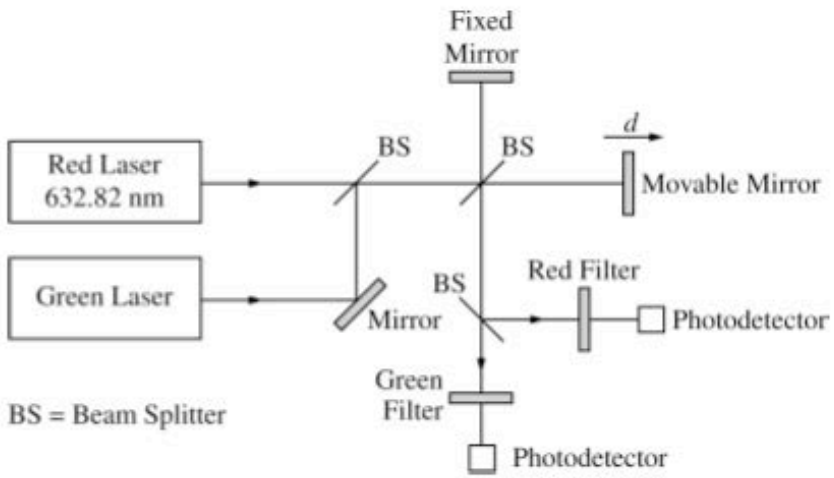
(A) 4 nF
(B) 40 nF
(C) 4 μF
(D) 40 μF
(E) 400 μF



40. In the circuit shown above, the switch S is closed at $t = 0$. Which of the following best represents the voltage across the inductor, as seen on an oscilloscope?



Inductors respond to CHANGE in current, so they reach highest voltage
the moment the switch is flipped. Then they drop off. And it happens pretty fast, by human standards…so not E.
Therefore D.



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What they are really asking: Do you know the wavelength of green light?