(b) Now use the expansion for \( \frac{1}{|\textbf{r} - \textbf{r}'|} \) in spherical coordinates to rewrite \( G(\textbf{r}, \textbf{r}') \) in spherical coordinates, i.e., write \( G(r, \theta, \phi, r', \theta', \phi') \).

(c) Find the potential if there is a charge distribution inside the grounded hemisphere. The total charge is \( Q \), and the charge density in spherical coordinates is proportional to \( r' \) in the region \( 0 \leq r < d' \) and \( 0 \leq \theta \leq \frac{\pi}{2} \), where \( d \) is. Please be sure to include both the case \( r > d \) and the case \( r = d \).

(d) Find the induced surface charge on the inside of both the hemispherical surface and the planar one. What must the total induced surface charge be?

(e) Use your Green's function to find the additional term in the potential inside the hemisphere if the hemisphere is no longer grounded, but now the potential on the spherical surface is \( V \sin^2 \theta \) for \( 0 \leq \theta \leq \frac{\pi}{2} \), where \( V \) is a constant. You may leave integrals over Legendre polynomials in your answer.

\[ \int P_{\ell}(x) dx = 0, \text{ for } \ell \text{ even and } \ell \neq 0. \]

3. (20 points) Consider the region inside a cylinder of radius \( b \) and length \( L \), with its axis on the \( z \)-axis and one end on the \( x-y \) plane. The potential on the curved surface is zero, and it is also zero on the end where \( z=0 \). On the outer end, where \( z=L \), the potential is \( V \cos^2 \phi \), where \( V \) is a constant. Find the potential everywhere inside the cylinder. You may leave integrals over Bessel functions in your answer.

4. (20 points) Using cylindrical coordinates, find the Dirichlet Green's function for the outside of an infinitely long cylinder of radius \( a \). Make an expansion in terms of functions of \( \phi \) and \( z \), and treat the \( \rho \) coordinate differently.