Review - Variational Principle, Scattering Theory

Var.Princ.: \[ E_\text{g} \leq \langle \psi | H | \psi \rangle \leq \langle \psi | H | \psi \rangle \leq E_1 \] for any normalized function \( \psi \).

If \( \langle \psi | H | \psi \rangle = 0 \) then \( \langle \psi | H | \psi \rangle \leq E_1 \) is energy of 1st excited state.

Suffering:

\[ J = \frac{\hbar^2}{2m} \left( \psi^* \nabla \psi - \psi \nabla^* \psi \right) \]

\[ \sigma = \frac{2\pi}{4\mu} \int_0^\infty d\Omega \int d\Omega' \left| f(\theta) \right|^2 \sin \theta \, d\theta \]

Partial waves:

\[ f(\theta) = \frac{1}{k} \sum_{l=0}^\infty \frac{\sin \theta}{\sqrt{2l+1}} \left( -e^{i\eta} \right)^{l+1} \]

\( \sigma = \frac{4\pi}{k^2} \sum_{l=0}^\infty \left( 2l+1 \right) \sin^2 \theta \)

Consider phase shifts for \( l < kr_0 \), \( r_0 \) is range of potential.

Born approx. \( f(\theta) = -\frac{m}{2\pi \hbar^2} \int V(r) e^{-i\eta} \frac{dr}{r} \)

\[ f(\theta) = -\frac{2m}{\hbar^2} \int_0 \sin Kr \, dr \int V(r) \sin Kr \]

valid for weak interaction potential, also for large incident energies.

Particles in box, harmonic oscillator, hydrogen atom.