PHYSICS 9HE--SPRING, 2014: STUDY GUIDE
TO IMPORTANT IDEAS, EXPERIMENTS, CALCULATIONS
Lecture slides are the best guide to all these topics, together with assigned reading in the text, as well as supplementary reading, and assigned problems

Final Exam will be comprehensive over course, with ca. 1/3 on material up to midterm, and 2/3 on material after that

**Special Relativity**
- Know the postulates of Special Relativity
- Proper time, proper length
- Time dilation (muon decay, planes circling earth), length contraction
- Radiative decay: as relevant to muon experiment and later discussions of nuclear properties and particle physics (see below)
- Doppler shift for light and sound: application to Doppler radar, auto speed monitoring, expansion of universe
- Energy-mass relationships, including reactions in which mass change becomes energy released/taken in (incl. nuclear binding energies and reaction energies from Ch. 12)

**General Relativity**
- Know the postulate of General Relativity (Principle of Equivalence)
- Effect of gravity on light, including shift of frequency, gravitational lensing & dark matter
- Effect of gravity on time (watch confusion with time dilation in Special Relativity)
- Practical examples: GPS satellites for which both Special and General Relativity important; Mössbauer effect with both Doppler and Gen. Rel. present
- Black holes and the Schwarzschild radius
- Motion of the perihelion of Mercury
- Gravitational lensing and dark matter
- Gravity waves, the graviton

**Waves and Introductory Quantum Ideas**
From some basic experiments seen already in 9HC:
- Emission and absorption spectra
- Blackbody radiation:
  - Planck's model vs. Rayleigh-Jeans model
  - Planck Blackbody formula and its meaning
  - Stefan-Boltzmann formula and Wein displacement laws and their uses
- The photoelectric effect
- Photon energy and photon momentum (\(E = pc\) only for particles of zero rest mass like photon...or other particles at very high energy...so be careful!)
- X-ray production in an x-ray tube:
  - Via two processes: bremsstrahlung and core electronic transitions
  - Moseley correlations of x-ray energy with atomic number, electron screening, and effective nuclear charges
  - Selection rules in x-ray transitions (from later disc. of time dependence in QM)
- Elastic wavelike scattering of x-rays--Bragg diffraction of x-rays by atomic layers in crystals: \(n\lambda = 2dsin\theta\)
- Electron-positron pair production and positron emission tomography (see discussion at end of quarter)
- Emission and absorption spectra of atoms, including selection rules (from later disc. of time dependence in QM)
- De Broglie wavelength for a particle: \(\lambda = h/p\)
- Davisson-Germer diffraction of electrons by crystal surfaces near surfaces:
2d and 3d aspects
-The double-slit experiment with light and electrons: building up images by counting
-Conserving momentum and energy in emission of light and creation/annihilation of particles (e.g. free electron cannot absorb a photon in the photoelectric effect)

Basic Concepts and Mathematics of Quantum Mechanics, Including Fourier Transforms

-Complex variables and complex functions—brief review
-Superposition of waves, wave packets, phase and group velocities
-Fourier series and integrals; use of and relationship to the Uncertainty Principles
-Example of RC electrical circuit as a Fourier filter, passing only low frequencies
-Sets of orthogonal, or orthogonal and normalized = orthonormal functions as basis sets for describing any function, including wave functions
-The Heisenberg Uncertainty Principles in position-momentum and energy-time
-Wave packets, group and phase velocities, the Dirac delta function
-Know the postulates of Quantum Mechanics and how to use them, including the general rules for solving and understanding a Q.M. problem (see lecture slides)
-Meaning of eigenvalues and wave functions as sums of orthonormal sets of Eigenfunctions, as e.g. Fourier series
-The Correspondence Principle: as energy a/o quantum no. goes up, retrieve classical-like behavior
-Time-dependent and time-independent Schroedinger Equations and wave functions: first example of the use of separation of variables
-Other uses of separation of variables in solving time-independent Sch. Eqn.: 3D particle in a rigid box, hydrogenic atom, rigid rotor model for molecular rotation, separation of nuclear and electronic motions in molecules, and other q.m. problems
-Calculation of probability densities, expectation values, and dipole matrix elements of operators: make use of even-odd character (parity) and symmetry wherever you can to simplify this. E.g.- an overall odd integrand, when integrated over 
-\(-\infty < x < +\infty\) must be zero; the same is true in 3 dimensions, whether
-\(-\infty < x < +\infty, -\infty < y < +\infty, -\infty < z < +\infty\) or
-\(0 < r < \infty, 0 < \theta < \pi, 0 < \phi < 2\pi\)
-Estimation of uncertainties via semi-classical approximations and the Uncertainty Principle
-Eigenfunction/eigenvalue relationships for different wave functions, including Parity operator \(\hat{\Pi}\) defined to keep track of even and odd wavefunctions in potentials that are even:
-\(\hat{\Pi} \psi(x) = \psi(-x) = \pm 1 \psi(x),\) with eigenvalues of +1 (even \(\psi\)) and -1 (odd \(\psi\))
(E.g., see table on 1D problems in lecture slide)

Quantum Mechanics in 1 Dimension

-Types of solutions and boundary conditions arising for different potentials:
  Particle in a rigid box
  Particle in a soft box
  Harmonic oscillator and application to molecular vibrations, including Correspondence Principle limits for all three cases
-Extension of rigid box to 3D by separation of variables:
  Degeneracy = several independent \(\psi\)'s yield same energy
-Be able to sketch qualitative form of a wave function in an arbitrary potential, assuming that it is piecewise constant, and using wavevector (\(k\)) or decay constant (\(\kappa\)) and Correspondence Principle arguments (e.g., \(|\psi|^2\) is maximum where classical particle moves most slowly, and wavelength also larger in such regions)
-Tunneling effects: general form of wavefunctions and boundary conditions in different
regions and calculation of tunneling probability $T(E)$ for special
cases: high-wide barrier, alpha decay, tunnel diode, field emission, and scanning
tunneling microscope (see lecture slides)

**Hydrogenic Atom Wave Functions and Properties**

- Separable form in $r, \theta,$ and $\phi$: $
\psi_{n\ell m}(r, \theta, \phi) = R_n(r)\Theta_{\ell m}(\theta)\Phi_{m\phi}(\phi)$

- Quantum nos. $n$, $\ell$, and $m_{\ell}$ and their significance to energy, angular momentum, and
parity eigenvalues:

  **Energy**: $\hat{H}\psi_{n\ell m} = E_n\psi_{n\ell m}, E_n$ from Bohr formula = $-\frac{Z^2e^2}{8\pi\varepsilon_0 a_0} \frac{1}{n^2}$

  **Square of orbital angular momentum**: $\hat{L}_z^2\psi_{n\ell m} = \hbar^2 \ell(\ell + 1)\psi_{n\ell m}$

  **Z component of orbital angular momentum**: $\hat{L}_z\psi_{n\ell m} = \hbar m\psi_{n\ell m}$

  **Parity**: $\hat{P}\psi_{n\ell m}(r, \theta, \phi) = \psi_{n\ell m}(r, \pi - \theta, \phi + \pi) = (-1)^m \psi_{n\ell m}$

- Normalization and calculation of probabilities and expectation values, including
integration with proper volume element in spherical polar coordinates:

  $dV = r^2 dr \sin\theta d\theta d\phi$

- Use of radial probability distribution $P_{nl}(r) = 4\pi r^2 R_{nl}(r)$ to illustrate shell structure
associated with a given $n$ value, as well as different degrees of nuclear charge screening
by the other electrons in a multi-electron atom

- Conversion of complex functions for $m_{\ell} \neq 0$ into the real functions of chemistry via sum
and difference, e.g. $p_x, p_y, p_z$

- Graphical representation of different aspects of wave functions in 1D, 2D, and 3D, including radial and angular nodes

- Degeneracy of atomic $\psi$'s

- Vector model of quantization of space for orbital angular momentum

- Magnetic moment associated with orbital angular momentum

- Normal Zeeman effect: different degenerate $m_{\ell}$ levels split by magnetic field

- Selection rules for dipole-radiation transitions and application to atomic spectra, x-ray
spectra, and molecular rotational and vibrational excitations

**Spin Angular Momentum, Spin Magnetic Moment**

- Stern-Gerlach experiment and existence of spin $\hat{s}$: one last relativistic effect!

- Adds quantum no. $m_s = \pm 1/2$ and spin magnetic moment $\mu_s = -\frac{e}{m} \hat{s}$

- Additional eigenvalue properties with spin added are:

  **Square of spin angular momentum**: $\hat{S}^2\psi_{n\ell m s} = \hbar^2 \frac{1}{2}\left(\frac{1}{2} + 1\right)\psi_{n\ell m s} = \frac{3}{4} \hbar^2 \psi_{n\ell m s}$

  **Z component of spin angular momentum**: $\hat{S}_z\psi_{n\ell m s} = \hbar m_s\psi_{n\ell m s} = \pm \frac{\hbar}{2} \psi_{n\ell m s}$

- Vector model of quantization of space for spin angular momentum

- Spin-orbit coupling and vector model of quantization of space for total angular
momentum \( \mathbf{j} = \mathbf{\ell} + \mathbf{s} \), with new quantum nos. of \( n, \ell, m, m_s \), and

Square of total angular momentum: \( \mathbf{\hat{J}}^2 \psi_{n/\ell/m} = \hbar^2 (j + 1) \psi_{n/\ell/m} \), with \( j = 1/2, 3/2, 5/2, \ldots \)

Z component of spin angular momentum: \( \mathbf{\hat{J}}_z \psi_{n/\ell/m} = \hbar m \psi_{n/\ell/m} \), with \( m = -j, -j+1, \ldots, j-1, j \)

-Spin-orbit interaction as an internal magnetic field created by nuclear motion around electron→current loop, and energy splittings (e.g. sodium emission/absorption doublet)

### Wave Functions and Energies for Multielectron Atoms

- The essential points:
  - the electron has intrinsic spin: \( s = 1/2 \), and z component linked to \( m_s = \pm 1/2 \)
  - this leads to four total quantum nos. for non-relativistic electrons in atoms:
    \( n, \ell, m, m_s \)
  - the wave function for a set of identical particles cannot yield results which depend on our choice of particle labelling: e.g., probability density:
    \[
    |\psi(\ldots\mathbf{r}_1\ldots\mathbf{r}_2\ldots)|^2 = |\mathbf{\hat{P}}_{12}\psi(\ldots\mathbf{r}_1\ldots\mathbf{r}_2\ldots)|^2 = |\psi(\ldots\mathbf{r}_2\ldots\mathbf{r}_1\ldots)|^2,
    \]
    where \( \mathbf{\hat{P}}_{12} \) is the permutation operator which just interchanges the labels of all space and spin coordinates, thus "trading the places" of electrons 1 and 2 in \( \psi \).
  - all particles are "fermions" or "bosons", depending on sign change when any two labels are interchanged. i.e., all valid many-particle wave functions are eigenfunctions of \( \mathbf{\hat{P}}_{12} \), and all particles are in two groups:
    Fermions (electrons and all particles with half-integral spin = \( 1/2, 3/2, 5/2, \ldots \)):
      antisymmetric \( \psi \)’s with -1 eigenvalue: \( \mathbf{\hat{P}}_{12}\psi = -1\psi \).
    Bosons (photons, all particles with integral spin = \( 0, 1, 2, 3, \ldots \)):
      symmetric \( \psi \)’s with +1 eigenvalue: \( \mathbf{\hat{P}}_{12}\psi = +1\psi \).

- This leads to the Pauli Exclusion Principle for electrons (or all Fermions):
  No two electrons in a multielectron atom can have all of \( n, \ell, m, m_s \) (or \( n, l, j, m_j \)) the same. (If they do, then \( \psi \) is trivially zero!)

- And also: Electrons with the same spin orientation (up, up) or (down, down) are never found at the same point in space, lowering their Coulomb repulsion (the Fermi hole and the energy-lowering “exchange interaction”): explains magnetism and Hund’s First Rule which yields maximum spin for open-shell coupling in atoms

- Inner-shell electron screening in multi-electron atoms lifts the H-atom degeneracy in \( \ell \) through an effective Z that varies with \( \ell \) for a given \( n \): energies always go as \( s < p < d < f \), with overlap of \( n \) values which e.g. makes 4s fill before 3d

- Pauli Principle then leads to filling of atomic levels in Periodic Table. If open shell at the end, Hund’s First Rule says fill with maximum total spin \( S \). (Don’t worry about other Rules.)

### Molecules

- “Electrons in a box” with one potential well on each atom; for diatomics—wave functions approximated as sums (bonding) and differences (anti-bonding) of atomic functions on each atom
- For polyatomic molecules, electronic wave functions describable as linear combinations of atomic orbitals on different atoms, with bonding/anti-bonding (nodal plane perpendicular to bond direction) character, and symmetry of \( \sigma \) (cylindrically symmetric around bond direction) or \( \pi \) (nodal plane through bond direction)
- Diatoms also behave like 1D harmonic oscillators in their vibrations, with quantum no. \( v \).
- Diatoms also behave like rigid “dumbbells” in their rotations: same angular wave functions as those of the atom, with angular momentum quantum nos. \( J \) and \( M_J \).
- Examples of: complex spectra showing simultaneously electronic, vibrational and rotational excitations, CO microwave absorption and ultraviolet photoelectron excitation, the microwave oven, and possible effects of cellphones on brain metabolism rate (via \( \Delta T \)?)

**Maxwell Boltzmann, Fermi-Dirac and Bose-Einstein Statistics (Review)**

- MB, FD, and BE statistics, basic formulas, similarities and differences
- Blackbody radiation as BE example, including solar spectrum, Cosmic microwave background.
- Electrons as FD example (see below in solids)

**Lasers and Holography**

- General principles of stimulated emission and population inversion (qualitative)
- Coherence (high monochromaticity and narrow angular spread) of laser light and application to holography (qualitative)
- Young’s double slit interference as a simple example of a hologram which can yield a holographic image

**Solids and Solid State Devices**

- Electron counting and level filling in solids
- Free-electron-like solids: density of states, Fermi energy, Fermi wave vector and velocity
- Li and Al as examples of nearly free-electron solids
- Periodic solids and Bloch functions as “universal” wave function form for solids
- The Kronig-Penney model and energy bands
- Semiconductor energy bands—e.g. Ge
- Metallic energy bands—e.g. non-magnetic Cu and ferromagnetic Fe
- Phenomenology of ferromagnetism and superconductivity/Cooper pairs
- Ionic solids and the Madelung sum
- Ionic energy bands—e.g. NaCl
- Semiconductor doping of n or p type
- The p-n junction diode, extension to the LED and the photovoltaic cell
- The LED-based laser
- The metal-oxide-semiconductor field-effect transistor (MOSFET)
- Transistors and the logic gates of IT devices
- Nanoscience and nanotechnology
- Moore’s Law for integrated circuits; analogue for magnetic bit density in storage media

**Nuclear Properties and Applications**

- Decay of nuclei and particles and their associated half lives
- Particles and anti-particles, e.g. electron and positron
- Notation for nuclides, including Z, N, and A and chemical symbol X: \( ^AX_N \)
- Nuclear sizes, charge density versus radius, Rutherford scattering (review from 9HC and Chapter 4—two basic equations)
- Nuclear binding energies per nucleon, calculation of and stability versus mass number A
- The nuclear interactions: proton-proton Coulomb barrier plus strong nucleon-nucleon interaction mediated by pion plus hard-core repulsion
- Shell model (may give net spin on nucleus) and liquid drop model for calculating binding energies per nucleon
- Basic nuclear decay modes and meaning for changes in \( ^AX_N \)
- Applications to carbon dating, nuclear energy production by fission and fusion, understanding radiation from radon in earth’s crust
- Enrichment of U by diffusion and centrifugation, breeder reactors using Pu
- Nuclear spin angular momentum, with quantum nos. \( I \) and \( M_I \), magnetic resonance (qualitative), Magnetic Resonance Imaging (MRI), in comparison to Positron Emission Tomography (PET)

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<thead>
<tr>
<th>Elementary Particles and Interactions</th>
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<tr>
<td>Leptons and Quarks, the table of elementary particles in the Standard Model and the manner in which quarks go together to make up common Hadrons = mesons (2 quarks, e.g. pion) + baryons (3 quarks, e.g., protons and neutrons)</td>
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<td>Four fundamental forces: electromagnetic, weak, strong, and gravitational, and four Bosons mediating them (photon; W,Z bosons; gluons; and gravitons, respectively)</td>
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<td>The Higgs Boson as a final ingredient that produces the mass of all of the particles</td>
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<td>Dark matter (e.g. from gravitational lensing-covered earlier) and dark energy (e.g. from increase in rate of universe expansion)</td>
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