Physics 9HE—Modern Physics  
Quiz 2  
27 February, 2014  
(100 points total)  
You may tear off this sheet.

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Miscellaneous data and equations:

c = 3.00 x 10^8 m/s  
e = 1.60 x 10^{-19} C  
1 eV = 1.60 x 10^{-19} J  
1 Å = 10^{-10} m

\[ M_{\text{Sun}} = 2 \times 10^{30} \text{ kg} \quad M_{\text{Earth}} = 5.98 \times 10^{24} \text{ kg} \quad r_{\text{Earth}} = 6.38 \times 10^6 \text{ m} \]

\[ m_e = 9.1094 \times 10^{-31} \text{ kg} \quad m_p = 1.6726 \times 10^{-27} \text{ kg} \quad m_n = 1.6749 \times 10^{-27} \text{ kg} \quad m(T) = 1.0078 \text{ u} \]

\[ G = 6.67 \times 10^{-11} \text{ Nt-m}^2/\text{kg}^2 \quad g = 9.81 \text{ m/s}^2 \quad \sigma = 5.67 \times 10^{-8} \text{ W} \cdot \text{m}^{-2} \cdot \text{K}^{-4} \]

\[ h = 6.63 \times 10^{-34} \text{ J-s} \quad \hbar = h/2\pi = 1.05 \times 10^{-34} \text{ J-s} \quad k_B = 1.38 \times 10^{-23} \text{ J-K}^{-1} \quad a_0 = 0.529 \text{ Å} \]

\[ N(t) = N_0 \exp(-t/\tau_0) = N_0 \exp(-0.693l/t_1/2) \quad k_c = 1/(4\pi \varepsilon_0) = 8.98 \times 10^9 \text{ N-m}^2/\text{C}^2 \quad R_H = 1.09678 \times 10^7 \text{ m}^{-1} \]

\[ \gamma = \frac{1}{(1 - v^2/c^2)^{1/2}} = 1 + 0.5 \beta^2 \quad v < c \quad \beta = v/c = [(v^2 - 1)/v^2]^{1/2} \]

\[ T = \gamma T_0 \quad L = L_0/\gamma \quad v = \nu_0 \frac{(1 \pm \beta)^{1/2}}{(1 \mp \beta)^{1/2}} = \nu_0 [1 \pm \beta] \quad \nu = c/\lambda \quad \nu \text{ (lect.)} = f(\text{book}) \]

\[ \dot{p} = \gamma m \dot{v} \quad E = \gamma mc^2 = K + mc^2 \quad E^2 = p^2 c^2 + m^2 c^4 \]

\[ E = pc = h\nu \quad \Delta \nu / \nu = \Delta T / T = -GM \left[ \frac{1}{r_1} - \frac{1}{r_2} \right] \approx -\frac{gH}{c^2} \quad \frac{\Delta \nu}{\nu} \approx -\frac{\Delta \lambda}{\lambda} \quad \text{if } \Delta \nu / \nu \ll 1 \quad R_{\text{Sch}} = \frac{2GM}{c^2} \]

\[ \lambda_{\text{max}} T = 2.898 \times 10^{-3} \text{ m-K} \quad l(\lambda, T) = \frac{2\pi c^2 h}{\lambda^5} \left[ \frac{1}{e^{\lambda / kT} - 1} \right] \quad R(T) = \varepsilon \sigma T^4 \]

\[ h\nu = K_{\text{max}} + \phi \quad n\lambda = 2 \text{dsin} \theta \quad F_{\text{cont}} = k_c \frac{q_1 q_2}{r^2} \quad F_{\text{radiat}} = \frac{mv^2}{r} \quad a_0 = \frac{4\pi \varepsilon_0 h^2}{mc^2} = 0.529 \text{ Å} \]

\[ \lambda = h/p \quad p = mk \quad \Delta x \Delta p_x \geq \hbar/2 \quad \Delta E \Delta t \geq \hbar/2 \quad v_{\text{ph}} = \omega/k \quad v_{\text{gr}} = d\omega/dk \]

\[ e^{i\kappa x} = \cos \kappa x + i \sin \kappa x \quad \cos x = \frac{1}{2} \left[ e^{ix} + e^{-ix} \right] \quad \sin x = \frac{1}{2i} \left[ e^{ix} - e^{-ix} \right] \]

\[ \sin 2t = 2 \sin t \cos t \quad \cos 2t = \cos^2 t - \sin^2 t = 2 \cos^2 t - 1 = 1 - 2 \sin^2 t \quad \psi(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos \left( \frac{2\pi n}{\ell} x \right) + \sum_{n=1}^{\infty} b_n \sin \left( \frac{2\pi n}{\ell} x \right) \quad \text{with } k_n = \frac{2\pi n}{\ell} \quad \text{and} \]

\[ a_0 = \text{aver. of } \psi \text{ over } \ell, \quad a_n = \frac{2}{\ell} \int_0^\ell \psi(x') \cos \left( \frac{2\pi n}{\ell} x' \right) dx', \quad b_n = \frac{2}{\ell} \int_0^\ell \psi(x') \sin \left( \frac{2\pi n}{\ell} x' \right) dx' \]

\[ \psi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} c(k) e^{ikx} dk, \quad \text{with } c(k) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \psi(x') e^{-ikx'} dx' \]

\[ \hat{H} \Psi = i\frac{\partial \Psi}{\partial t} \quad \hat{H} = \hat{K} + V \quad \Psi = \psi e^{iEt/\hbar} = e^{-i\omega t} \quad \hat{H} \Psi = E \Psi \]

\[ \hat{p}_x = -i\hbar \frac{\partial}{\partial x} \quad \hat{K}_x = -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} \quad \Delta A = \sqrt{\langle A^2 \rangle - \langle A \rangle^2} < A > = \int \psi^* \hat{A} \psi dx \quad \hat{A}_\alpha = a\psi_a \]
\[ k = \sqrt{\frac{2m(E-V)}{\hbar^2}} \quad \psi \propto e^{ikx} \quad \psi \propto \sin kx, \cos kx \quad \kappa = \alpha = \frac{1}{\delta} = \sqrt{\frac{2m(V-E)}{\hbar^2}} \quad \psi \propto e^{ikx} \]

\[ \psi_n = \left( \frac{2}{L} \right)^{1/2} \sin \left( \frac{n\pi x}{L} \right) \quad E_n = \frac{\pi^2 \hbar^2 n^2}{2mL^2} \]

\[ \psi_n = H_n(x)e^{-ax^2/2} \quad \alpha = \sqrt{\frac{m\kappa}{\hbar^2}} \quad \omega = \sqrt{\frac{k}{m}} \quad E_n = \left( n + \frac{1}{2} \right) \hbar \omega \]

\[ \psi_j = Y_{am}(\theta,\phi) = \Theta_{am}(\theta)\Phi_{am}(\phi) \quad E_j = \frac{\hbar^2 J(J+1)}{2l} \quad l = \frac{m_m z - m_z R_{ss}}{m_s + m_z} \]

\[ T = \left[ 1 + \frac{V_0^2 \sin^2(\kappa L)}{4E(E-V_0)} \right]^{-1} \quad T \approx \frac{16E}{V_0} \left[ 1 - \frac{E}{V_0} \right] e^{-2\kappa L} \quad \text{(when } \kappa L \gg 1) \]

\[ T_{FE} \approx \exp \left[ \frac{4\phi^{3/2}2m_e}{3e\hbar^2} \left( \frac{2m_e}{eV} \right) \right] \quad l_{STM} \approx e^{-2KL} \]

\[ \lambda_a = f_{col} T_a \quad T_a = \exp \left[ -4\pi Z \sqrt{\frac{0.0993 \text{ MeV}}{E_a(\text{MeV})}} + 8 \sqrt{\frac{Z R_{\text{nuc}}(m)}} \right] \]

\[ F_{\text{coul}} = k \frac{q_1 q_2}{r^2} \quad F_{\text{radial}} = \frac{mv^2}{r} \quad E_n = -\frac{Z^2 e^2}{8\pi \epsilon_0 m_r n^3} = -\frac{136 Z^2}{n^2} (eV) \quad r_n = \frac{4\pi \epsilon_0 \hbar^2}{m_e Z e^2} n^2 = a_0 \frac{n^2}{Z} \]

\[ a_0 = \frac{4\pi \epsilon_0 \hbar^2}{m_e e^2} = 0.529 \ \text{Å} \quad \nu_n = \frac{nh}{m_r r_n} \quad \frac{1}{\lambda} = Z^2 R_{ss} \left[ \frac{1}{n^2} - \frac{1}{n_v^2} \right] \quad \mu_e = m_e \left[ \frac{M}{M + m} \right] \]

\[ x = r \sin \theta \cos \phi \quad y = r \sin \theta \sin \phi \quad z = r \cos \theta \quad dV = r^2 \sin \theta \, dr \, d\theta \, d\phi \]

\[ \psi_{n_m}(r,\theta,\phi) = R_{n}(r) \Theta_{nm}(\theta) \Phi_{nm}(\phi) \quad P_{n_m}(r) = r^2 R_{n_m}(r) \quad \int \propto \left| \frac{\psi_{\text{final}}(\hat{r})}{\psi_{\text{initial}}(\hat{r})} \right|^2 \]

\[ \mu = iA \quad E = -\bar{\mu} \cdot \bar{B} \quad \bar{\mu} = -\frac{e}{2m} \frac{\vec{L}}{\hbar} = -\mu_B \frac{\hat{L}}{\hbar} \quad \bar{s} = -\frac{e}{m} \hat{s} = -2 \mu_B \frac{\hat{s}}{\hbar} \]

\[ \mu_B = \frac{e\hbar}{2m_e} = 9.274 \times 10^{-24} \text{ J/Tesla} \]

\[ \psi_{j}^{\text{MOC}}(\hat{r}) = \sum_{A_i} c_{A_i} \phi_{A_i}(\hat{r}) \]

---Tear off this sheet and begin exam---
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Name (printed)_______________________________________
Name (signature)_______________________________________
Student ID No._________________________________________

Yes ___ No___ would like to go on the tour of LBNL on 8 March

If Yes, I would like to bring ____ guests

If Yes, I Do___Do Not___have transportation, and

If Yes, I have/can arrange a vehicle Yes___No___

If yes, I will have room for _____people

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[1] (25 points)
Would the following be a suitable approximate wavefunction for two neutrons with spin \( \frac{1}{2} \) which overlap in space, where a and b represent two different sets of quantum numbers including the neutron spin and C is a suitable normalization constant?

\[
\Psi(\vec{r}_1, \vec{s}_1; \vec{r}_2, \vec{s}_2) = C[\Phi_a(\vec{r}_1, \vec{s}_1)\Phi_b(\vec{r}_2, \vec{s}_2) + \Phi_b(\vec{r}_1, \vec{s}_1)\Phi_a(\vec{r}_2, \vec{s}_2)]
\]

Explain why or why not using the permutation operator.

\[
\Psi(\vec{r}_1, m_{s_1}, \vec{r}_2, m_{s_2}) = \hat{\tau}_{12}[\phi_a(\vec{r}_1, m_{s_1})\phi_b(\vec{r}_2, m_{s_2}) + \phi_b(\vec{r}_1, m_{s_1})\phi_a(\vec{r}_2, m_{s_2})] = [\phi_a(\vec{r}_2, m_{s_2})\phi_b(\vec{r}_1, m_{s_1}) + \phi_b(\vec{r}_2, m_{s_2})\phi_a(\vec{r}_1, m_{s_1})] = + \Psi(\vec{r}_1, m_{s_1}, \vec{r}_2, m_{s_2}) \Rightarrow \text{SYMMETRIC, SO NOT OK FOR NEUTRONS}
\]
[2] (50 points)
The manganese atom has 25 electrons.
(a) Write down its complete electronic configuration.

\[ 1s^22s^22p^63s^23p^63d^54s^2 \]

(b) In a photoelectric effect experiment, the Mn 2p energies are found to be split into two separate levels. What is the origin of this splitting?

Spin-orbit splitting, so 2p (\( \ell = 1 \)) splits into \( j = 2p_{1/2} \) and \( 2p_{3/2} \)

(c) What would be the total spin quantum number \( S \) of the manganese atom in its ground state?

The five 3d electrons will all couple with their spins in the same direction so one can write:
\( 3d^5 \uparrow\uparrow\uparrow\uparrow\uparrow \) and \( S = 5/2 \)

(d) Treating the total spin as a typical quantum mechanical angular momentum, and noting that the total orbital angular momentum \( L \) would be zero for this case, how many total states would the manganese atom have in a magnetic field, specify the quantum number involved, and give the eigenvalues associated with it.

6 states, of \( m_s = 5/2, 3/2, 1/2, -1/2, -3/2, -5/2 \), and a vector diagram as below,
and eigenvalues of $S^2 = \hbar^2 5/2(5/2+1) = \hbar^2 35/4$, and $S_z = \hbar m_s$.

(e) If the manganese atom were placed in a strong magnetic field of 1 Tesla along the $z$ axis, what would the energy spacing be between adjacent levels?

Just use these given equations, and note that $B$ is along $z$, so the dot product just picks up the $z$ component, and with some given equations we have

$$E = -\mu_s \cdot \vec{B} = -\mu_s \vec{z} = -2 \mu_B \frac{\hat{z}}{\hbar}$$

$$\mu_B = \frac{e\hbar}{2m_e} = 9.274 \times 10^{-24} \text{ J/Tesla}$$

which combine for our special case to give

$$E = -\mu_s \cdot \vec{B} = -\mu_s z B_z = -\left[ -2 \mu_B \frac{\hat{z}}{\hbar} \right] B_z = -2 m_B \frac{\hbar m_s}{\hbar} B_z = [2 \times 9.274 \times 10^{-24} \text{ J/Tesla}] (m_B B_z).$$

Difference between two adjacent $m_s$ values is just a dimensionless 1, so

$$\Delta E = [2 \times 9.274 \times 10^{-24} \text{ J/Tesla}] (1 \times 1) = 1.8548 \times 10^{-33} \text{ J}$$

[3] [25 Points]
The iso-probability contour of one of the molecular electronic wavefunctions of CO is shown below, with the sign of the wavefunction also indicated. Show with a sketch and an equation the approximate atomic orbital makeup of this wave function, and indicate whether it is bonding on anti-bonding.

More than you ever wanted to know here is from Slide 20 of Set 6 (next page), with the equation for what is called $1\pi_x$ here being

$$\varphi_{1\pi_x} \approx C_{C_2p_x} \varphi_{C_2p_x} + C_{O_2p_x} \varphi_{O_2p_x} \quad \text{and} \quad C_{O_2p_x} \text{ clearly greater than } C_{C_2p_x}$$
The LCAO or tight-binding picture for CO:

\[ \varphi_j^{MO}(\vec{r}) = \sum \alpha_{i,j} \varphi_{Ai}(\vec{r}) \]

---End of examination---