2. a) Given the placement of the \(-\) sign, it moves in the \(+x\)-direction.
   
b) By the same reasoning as in (a), it moves in the \(-x\)-direction.
   
c) It is a complex number.
   
d) It moves in the \(+x\)-direction. Looking at a particular phase \(kx - \omega t\), \(x\) must increase as \(t\) increases in order to keep the phase constant.

* 5.

\[
\Psi^* \Psi = A^2 r^2 \exp \left( \frac{-2r}{\alpha} \right)
\]

\[
\int_0^\infty \Psi^* \Psi \, dr = A^2 \int_0^\infty r^2 \exp \left( \frac{-2r}{\alpha} \right) \, dr = A^2 \left[ \frac{2}{(2/\alpha)^3} \right] = \frac{A^2 \alpha^3}{4} = 1
\]

Therefore

\[
\Lambda = \sqrt{\frac{4}{\alpha^3}} = 2\alpha^{-3/2}
\]

7. a) The wave function does not satisfy condition 3. The derivative of the wave function is not continuous at \(x = 0\).

b) Based on (a) the wave function cannot be realized physically.

c) Very close to \(x = 0\) we could modify the function so that its derivative is continuous. If we do so just in the neighborhood of \(x = 0\), we need not change the function elsewhere.

8.

\[
\bar{x} = \frac{3.4 + 3.9 + 5.2 + 4.7 + 4.1 + 3.8 + 3.9 + 4.7 + 4.1 + 4.5 + 3.8 + 4.5 + 4.8 + 3.9 + 4.4}{15} = 4.247
\]

\[
\langle x^2 \rangle = \frac{3.4^2 + 3.9^2 + 5.2^2 + 4.7^2 + 4.1^2 + 3.8^2 + 3.9^2 + 4.7^2 + 4.1^2 + 4.5^2 + 3.8^2 + 4.5^2 + 4.8^2 + 3.9^2 + 4.4^2}{15}
\]

\[
\langle x^2 \rangle = 18.254
\]

The standard deviation is

\[
\sigma = \sqrt{\frac{\sum (x_i - \bar{x})^2}{N}} = \sqrt{\frac{\sum (x_i^2 - 2x_i \bar{x} + \bar{x}^2)}{N}} = \sqrt{\frac{\sum x_i^2}{N} - 2\bar{x} \sum x_i/N + \sum \bar{x}^2/N}
\]

Look at the three terms in the sum. The first is just \(\langle x^2 \rangle\). The second is \(-2\bar{x} \langle \bar{x} \rangle = -2\bar{x}^2\). The third term is

\[
\frac{\sum \bar{x}^2}{N} = \frac{N \bar{x}^2}{N} = \bar{x}^2
\]

Putting the results together

\[
\sigma = \sqrt{\langle x^2 \rangle - 2\bar{x} \langle \bar{x} \rangle + \bar{x}^2} = \sqrt{\langle x^2 \rangle - \bar{x}^2}
\]

For the data given we have

\[
\sigma = \sqrt{18.254 - (4.247)^2} = 0.466
\]
9. If \( V \) is independent of time, then we can use the time-independent Schrödinger equation. Then by Equation (6.15)

\[
\Psi^* \Psi = \psi^*(x) \psi(x) e^{-i\omega t} e^{i\omega t} = \psi^*(x) \psi(x)
\]

Then

\[
\int \Psi^* \Psi \, dx = \int \psi^*(x) \psi(x) \, dx
\]

which is independent of time.

11.

\[
\int_0^\pi \psi^* \psi \, dx = A^2 \int_0^\pi \sin^2 \left( x \right) \, dx = A^2 \frac{\pi}{2} = 1
\]

so \( A = \sqrt{\frac{2}{\pi}} \) and the probability of being in the interval \([0, \pi/4]\) is

\[
P^* = \int_0^{\pi/4} \psi^* \psi \, dx = \frac{2}{\pi} \int_0^{\pi/4} \sin^2 \left( x \right) \, dx = \frac{2}{\pi} \left( \frac{x}{2} - \frac{1}{4} \sin(2x) \right) \bigg|_0^{\pi/4}
\]

\[
= \frac{2}{\pi} \left( \frac{\pi}{8} - \frac{1}{4} \right) = \frac{1}{4} - \frac{1}{2\pi} = 0.091
\]

* 15. a) Starting with Equation (6.35) and using the electron mass and the length given, we have

\[
E_n = n^2 \frac{\pi^2 \hbar^2}{2mL^2} = n^2 \frac{\pi^2 (hc)^2}{2 (me^2) L^2}
\]

\[
= n^2 \frac{\pi^2 (107.3 \text{ eV} \cdot \text{nm})^2}{2 (5.11 \times 10^6 \text{ eV})(2000 \text{ nm})^2} = n^2 \left( 9.40 \times 10^{-8} \text{ eV} \right)
\]

Then the three lowest energy levels are: \( E_1 = 9.40 \times 10^{-8} \text{ eV} \); \( E_2 = 1.88 \times 10^{-7} \text{ eV} \); and \( E_1 = 2.82 \times 10^{-7} \text{ eV} \);

b) Average kinetic energy equals \( \frac{3}{2} kT = \frac{3}{2} \left( 1.381 \times 10^{-23} \text{ J/K} \right) \left( \frac{1 \text{ eV}}{1.602 \times 10^{-19} \text{ J}} \right) \) 13K

which equals \( 1.68 \times 10^{-3} \text{ eV} \). Substitute this value into the equation above as \( E_n \) and solve for \( n \). We find \( n = 134 \).

20. a) As in the previous problem

\[
E_1 = \frac{\hbar^2 c^2}{8 m c^2 L^2} = \frac{(1240 \text{ eV} \cdot \text{nm})^2}{8 (938.27 \times 10^6 \text{ eV})(1.4 \times 10^{-5} \text{ nm})^2} = 1.05 \text{ MeV}
\]

b)

\[
E_1 = \frac{\hbar^2 c^2}{8 m c^2 L^2} = \frac{(1240 \text{ eV} \cdot \text{nm})^2}{8 (3727 \times 10^6 \text{ eV})(1.4 \times 10^{-5} \text{ nm})^2} = 263 \text{ keV}
\]

23. a) The wavelengths are longer for the finite well, because the wave functions can leak outside the box.

b) Generally shorter wavelengths correspond to higher energies, so we expect energies to the lower for the finite well.

c) Generally the number of bound states is limited by the depth of the well. We expect no bound states for \( E > V_0 \).
\[ E = \frac{\pi^2 \hbar^2}{2mL^2} (n_1^2 + n_2^2 + n_3^2) = E_0 (n_1^2 + n_2^2 + n_3^2) \]

where
\[ E_0 = \frac{\pi^2 \hbar^2}{2mL^2} \]

Then the second, third, fourth, and fifth levels are
\[
\begin{align*}
E_2 &= (2^2 + 1^2 + 1^2) E_0 = 6E_0 & \text{(degenerate)} \\
E_3 &= (2^2 + 2^2 + 1^2) E_0 = 9E_0 & \text{(degenerate)} \\
E_4 &= (3^2 + 1^2 + 1^2) E_0 = 11E_0 & \text{(degenerate)} \\
E_5 &= (2^2 + 2^2 + 2^2) E_0 = 12E_0 & \text{(not degenerate)}
\end{align*}
\]

*28. We must normalize by evaluating the triple integral of \( \psi^* \psi \):
\[
\iiint \psi^* \psi \, dx \, dy \, dz = 1
\]

with \( \psi(x, y, z) \) given by Equation (6.47) in the text. We can evaluate the iterated triple integral
\[
A^2 \int_0^L \sin^2 \left( \frac{\pi x}{L} \right) \, dx \int_0^L \sin^2 \left( \frac{\pi y}{L} \right) \, dy \int_0^L \sin^2 \left( \frac{\pi z}{L} \right) \, dz = A^2 \left( \frac{L}{2} \right)^3 = 1
\]

Solving for \( A \) we find
\[
A = \left( \frac{2}{L} \right)^{3/2}
\]

32. Normalization
\[
1 = \int_{-\infty}^{\infty} \psi^* \psi \, dx = A^2 \int_{-\infty}^{\infty} x^2 e^{-ax^2} \, dx = 2A^2 \int_0^{\infty} x^2 e^{-ax^2} \, dx = 2A^2 \frac{1}{4a} \sqrt{\frac{\pi}{a}}
\]

37. The classical frequency for a two-particle oscillator is (see Chapter 10, Equation (10.4))
\[
\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{k}{m_1 + m_2}} = \sqrt{\frac{2k}{m}} \quad \text{since the masses are equal in this case.}
\]
The energies of the ground state \( E_0 \) and the first three excited states are given by \( E_n = \left( n + \frac{1}{2} \right) \hbar \omega \) so the possible transitions (from \( E_3 \) to \( E_2 \), \( E_3 \) to \( E_1 \), etc. are \( \Delta E = \hbar \omega, 2\hbar \omega, \) and \( 3\hbar \omega \), or
\[
\hbar \omega = \hbar \sqrt{\frac{2k}{m}} = (6.582 \times 10^{-16} \text{ eV} \cdot \text{s}) \sqrt{\frac{2 (1.1 \times 10^3 \text{ N/m})}{1.673 \times 10^{-27} \text{ kg}}} = 0.755 \text{ eV}
\]
\[
\lambda = \frac{hc}{E} = \frac{1240 \text{ eV} \cdot \text{nm}}{0.755 \text{ eV}} = 1640 \text{ nm}
\]
\[
2\hbar \omega = 2 (6.582 \times 10^{-16} \text{ eV} \cdot \text{s}) \sqrt{\frac{2 (1.1 \times 10^3 \text{ N/m})}{1.673 \times 10^{-27} \text{ kg}}} = 1.51 \text{ eV}
\]
\[
\lambda = \frac{hc}{E} = \frac{1240 \text{ eV} \cdot \text{nm}}{1.51 \text{ eV}} = 821 \text{ nm}
\]
\[
3\hbar \omega = 3 (6.582 \times 10^{-16} \text{ eV} \cdot \text{s}) \sqrt{\frac{2 (1.1 \times 10^3 \text{ N/m})}{1.673 \times 10^{-27} \text{ kg}}} = 2.26 \text{ eV}
\]
\[
\lambda = \frac{hc}{E} = \frac{1240 \text{ eV} \cdot \text{nm}}{2.26 \text{ eV}} = 549 \text{ nm}
\]
40. In each case $\kappa L \gg 1$ so we can use

$$T = 16 \frac{E}{V_0} \left( 1 - \frac{E}{V_0} \right) e^{-2\kappa L}$$

where

$$\kappa = \sqrt{\frac{2m_e^2}{h^2}} (V_0 - E) = \frac{2 \left( 3727 \times 10^6 \text{ eV} \right) \left( 10 \times 10^6 \text{ eV} \right)}{197.4 \text{ eV} \cdot \text{nm}} = 1.38 \times 10^{15} \text{ m}^{-1}$$

a) With $L = 1.3 \times 10^{-14} \text{ m}$

$$T_a = 16 \frac{5 \text{ MeV}}{15 \text{ MeV}} \left( 1 - \frac{5 \text{ MeV}}{15 \text{ MeV}} \right) e^{-2 \left( 1.38 \times 10^{15} \text{ m}^{-1} \right) \left( 1.3 \times 10^{-14} \text{ m} \right)} = 9.3 \times 10^{-16}$$

b) With $V_0 = 30 \text{ MeV}$

$$\kappa = \sqrt{\frac{2m_e^2}{h^2}} (V_0 - E) = \frac{2 \left( 3727 \times 10^6 \text{ eV} \right) \left( 25 \times 10^6 \text{ eV} \right)}{197.4 \text{ eV} \cdot \text{nm}} = 2.19 \times 10^{15} \text{ m}^{-1}$$

$$T_b = 16 \frac{5 \text{ MeV}}{30 \text{ MeV}} \left( 1 - \frac{5 \text{ MeV}}{30 \text{ MeV}} \right) e^{-2 \left( 2.19 \times 10^{16} \text{ m}^{-1} \right) \left( 1.3 \times 10^{-14} \text{ m} \right)} = 4.2 \times 10^{-25}$$

c) With $V_0 = 15 \text{ MeV}$ we return to the original value of $\kappa$, but now $L = 2.6 \times 10^{-14} \text{ m}$ and

$$T_c = 16 \frac{5 \text{ MeV}}{15 \text{ MeV}} \left( 1 - \frac{5 \text{ MeV}}{15 \text{ MeV}} \right) e^{-2 \left( 1.38 \times 10^{15} \text{ m}^{-1} \right) \left( 2.6 \times 10^{-14} \text{ m} \right)} = 2.4 \times 10^{-31}$$

By comparison $T_a > T_b > T_c$.

41. When $E > V$ the wave function is oscillating, with a longer wavelength as $E - V$ decreases. Then when $E < V$ the wave function decays.
As in the text we find
\[ E = \frac{\hbar^2\pi^2}{2m} \left( \frac{n_1^2}{L_1^2} + \frac{n_2^2}{L_2^2} + \frac{n_3^2}{L_3^2} \right) \]
and substituting the given values of \( L \), we find
\[ E = \frac{\hbar^2\pi^2}{2mL^2} \left( n_1^2 + 2n_2^2 + \frac{n_3^2}{4} \right) \]

Letting \( E_0 = \frac{\hbar^2\pi^2}{2mL^2} \) we have
\[ E_1 = E_0 \left( 1 + 2 + \frac{1}{4} \right) = \frac{13}{4} E_0 \]
\[ E_2 = E_0 \left( 1 + 2 + \frac{2^2}{4} \right) = 4E_0 \]
\[ E_3 = E_0 \left( 1 + 2 + \frac{3^2}{4} \right) = \frac{21}{4} E_0 \]
\[ E_4 = E_0 \left( 2^2 + 2 + \frac{1}{4} \right) = \frac{25}{4} E_0 \]
\[ E_5 = E_0 \left( 1 + 2 + \frac{4^2}{4} \right) = E_0 \left( 2^2 + 2 + \frac{2^2}{4} \right) = 7E_0 \]

Of those listed, only \( E_5 \) is degenerate.

Using the known values of \( \psi_1 \) and \( \psi_2 \) we see
\[ \psi = \frac{1}{2} \psi_1 + \frac{\sqrt{3}}{2} \psi_2 = \frac{1}{2} \sqrt{\frac{2}{L}} \sin \left( \frac{\pi x}{L} \right) + \frac{\sqrt{3}}{2} \sqrt{\frac{2}{L}} \sin \left( \frac{2\pi x}{L} \right) \]
\[ \psi = \sqrt{\frac{1}{2L}} \sin \left( \frac{\pi x}{L} \right) + \sqrt{\frac{3}{2L}} \sin \left( \frac{2\pi x}{L} \right) \]

For normalization
\[ \int_0^L \psi^* \psi \, dx = \int_0^L \frac{1}{2L} \sin^2 \left( \frac{\pi x}{L} \right) + \frac{3}{2L} \sin^2 \left( \frac{2\pi x}{L} \right) \, dx \]

The third term vanishes because of the orthogonality of the trig functions, leaving
\[ \int_0^L \psi^* \psi \, dx = \int_0^L \frac{1}{2L} \sin^2 \left( \frac{\pi x}{L} \right) + \frac{3}{2L} \sin^2 \left( \frac{2\pi x}{L} \right) \, dx \]
\[ = \frac{1}{2L} \int_0^L \sin^2 \left( \frac{\pi x}{L} \right) \, dx + \frac{3}{2L} \int_0^L \sin^2 \left( \frac{2\pi x}{L} \right) \, dx \]
\[ = \frac{1}{2L} \left( \frac{L}{2} \right) + \frac{3}{2L} \left( \frac{L}{2} \right) = 1 \text{ as required} \]