Physics 9HE-Modern Physics
Final Examination
March 22, 2012
(100 points total)
You may tear off this sheet.

---

Miscellaneous data and equations:

- $c = 3.00 \times 10^8 \text{ m/s}$
- $e = 1.60 \times 10^{-19} \text{ C}$
- $1 \text{ eV} = 1.60 \times 10^{-19} \text{ J}$
- $1 \text{ Å} = 10^{-10} \text{ m}$
- $M_{\text{Sun}} = 2 \times 10^{30} \text{ kg}$
- $M_{\text{Earth}} = 5.98 \times 10^{24} \text{ kg}$
- $r_{\text{Earth}} = 6.38 \times 10^6 \text{ m}$
- $m_e = 9.1094 \times 10^{-31} \text{ kg}$
- $m_0 = 0.5110 \text{ MeV/c}^2$
- $m_p = 1.6726 \times 10^{-27} \text{ kg}$
- $m_n = 1.6749 \times 10^{-27} \text{ kg}$
- $m_{(1H)} = 1.0078 \text{ u}$
- $G = 6.67 \times 10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2$
- $g = 9.81 \text{ m/s}^2$
- $\sigma = 5.67 \times 10^{-8} \text{ W}\cdot\text{m}^{-2}\cdot\text{K}^{-4}$
- $h = 6.63 \times 10^{-34} \text{ J}\cdot\text{s}$
- $\hbar = h/2\pi$
- $k_B = 1.38 \times 10^{-23} \text{ J}\cdot\text{K}^{-1}$
- $a_0 = 0.529 \text{ Å}$
- $N(t) = N_0 \exp(-t/\tau_0) = N_0 \exp(-0.693t/\tau_{1/2})$
- $k_c = 1/(4\pi\sigma_0) = 8.98 \times 10^9 \text{ N}\cdot\text{m}^2\cdot\text{C}^{-2}$
- $R_H = 1.09678 \times 10^{7} \text{ m}^{-1}$

\[ \gamma = \frac{1}{1-v^2/c^2} = \left(1 + \frac{\beta^2}{1-\beta^2}\right) \leq v \ll c \]  
\[ \beta = \frac{v}{c} = [(\gamma-1)/\gamma]^{1/2} \]

\[ T = \gamma T_0 \]

\[ L = L_0 \gamma^2 = v_0 \left(1 + \frac{\beta^2}{1-\beta^2}\right) \leq v_0 \]  
\( \beta < 1 \)

\[ \gamma = c/\lambda \]

\[ \nu = \nu_0 (1 + \frac{\beta}{1-\beta}) \leq \nu_0 [1 + \beta] \]  
for \( \beta < 1 \)

\[ \nu = \nu_{\text{lect.}} = f(\text{book}) \]

\[ \ddot{\rho} = \gamma \mu \dot{E} = \gamma mc^2 = K + mc^2 \]

\[ E = pc = \hbar \nu = \Delta T \]

\[ - \frac{\Delta T}{c^2} = \frac{GM}{c^2} \left[ \frac{1}{r_1} - \frac{1}{r_2} \right] \approx \frac{gH}{c^2} \]

\[ \frac{\Delta T}{c^2} \approx \frac{\Delta \lambda}{c^2} \left( \frac{\nu}{c} < 1 \right) \]

\[ R_{\text{Sch}} = \frac{2GM}{c^2} \]

\[ \lambda_{\text{max}} = 2.898 \times 10^{-3} \text{ m} \cdot K \]

\[ I(\lambda, T) = \frac{2\pi c^2 h}{\lambda} \left( \frac{1}{e^{\lambda c/c} - 1} \right) \]

\[ R(T) = \varepsilon \sigma T^4 \]

\[ h\nu = K_{\text{max}} + \phi \]

\[ n\lambda = 2d \sin \theta \]

\[ F_{\text{coul}} = k_c \frac{q_1 q_2}{r^2} \]

\[ F_{\text{radial}} = \frac{mv^2}{r} \]

\[ a_0 = \frac{4\pi\varepsilon_0 \hbar^2}{mc^2} = 0.529 \text{ Å} \]

\[ \lambda = h/p \quad p = \hbar k \]

\[ \Delta A = \Delta P_x \geq \hbar \Delta E \Delta t \geq \hbar/2 \]

\[ v_{ph} = \omega/\hbar, \quad v_{pr} = d\omega/dk \]

\[ e^{i\pi} = \cos x + \frac{1}{2} e^{ix} + e^{-ix} \quad \cos x = \frac{1}{2} \left[ e^{ix} + e^{-ix} \right] \]

\[ \sin 2t = 2 \sin t \cos t \quad \cos 2t = \cos^2 t - \sin^2 t = 2 \cos^2 t - 1 = 1 - 2 \sin^2 t \]

\[ \phi(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos \left( \frac{2\pi n}{\ell} x \right) + \sum_{n=1}^{\infty} b_n \sin \left( \frac{2\pi n}{\ell} x \right), \quad \text{with } k_n = \frac{2\pi n}{\ell}, \text{ and} \]

\[ a_0 = \text{aver of } \phi \text{ over } \ell, \quad a_n = \frac{2}{\ell} \int_{0}^{\ell} \phi(x) \cos \left( \frac{2\pi n}{\ell} x \right) dx' \quad \text{with } k_n = \frac{2\pi n}{\ell}, \text{ and} \]

\[ a_n = \int_{0}^{\infty} e^{-i\pi} c(k) e^{ikx} dx', \quad b_n = \int_{0}^{\infty} e^{-i\pi} c(k) e^{-ikx} dx' \]

\[ \psi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} c(k) e^{i\kappa x} dk, \quad \text{with } c(k) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \psi(x) e^{-i\kappa x} dx' \]

\[ \hat{H} \psi = i\hbar \frac{\partial \psi}{\partial t} \quad \hat{H} = \hat{K} + \hat{V} \quad \psi = \psi e^{-i\hat{V}/\hbar} = \psi e^{-i\hat{E}/\hbar} \quad \hat{H} \psi = E \psi \]

\[ \hat{p}_x = -i\hbar \frac{\partial}{\partial x} \quad \hat{K}_x = -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} \quad \Delta A = \sqrt{\langle A^2 \rangle - \langle A \rangle^2} \leq A \geq \int \psi^* \hat{A} \psi dx \quad \hat{A} \psi_a = a \psi_a \]
\[ k = \sqrt{\frac{2m(E-V)}{\hbar^2}} \quad \psi \propto e^{ikx} \quad \psi \propto \sin kx, \cos kx \quad \kappa = \alpha = \frac{1}{\delta} = \sqrt{\frac{2m(V-E)}{\hbar^2}} \quad \psi \propto e^{ix} \]

\[ \psi_n = \left( \frac{2}{L} \right)^{1/2} \sin \left( \frac{n\pi x}{L} \right) \quad E_n = \frac{n^2\hbar^2 n^2}{2mL^2} \]

\[ \psi_n = H_n(x) e^{-ax^2} \quad \alpha = \sqrt{\frac{mK}{\hbar^2}} \quad \omega = \frac{\kappa}{m} \quad E_n = \left(n + \frac{1}{2} \right) \hbar \omega \]

\[ \psi_j = \psi_{n_1} \psi_{n_2} \quad E_j = \frac{\hbar^2 J(J+1)}{2l} \quad l = \frac{m m_s}{m + m} \]

\[ T = \left[ 1 + \frac{V_0^2 \sin^2(kL)}{4E(E - V_0)} \right]^{-1} \quad T = \left[ 1 + \frac{V_0^2 \sin^2(kL)}{4E(V_0 - E)} \right]^{-1} \approx \frac{16 E}{V_0} \left[ 1 - \frac{E}{V_0} \right] e^{-2K} \text{ (when } kL \gg 1) \]

\[ T_{FE} \approx \exp \left[ -\frac{4\phi^{3/2}2m_e}{3e\hbar} \frac{2m_e}{dx} \right] \quad I_{STM} \propto e^{-2KL} \quad \lambda_a = f_{coll} f_a ; \quad T_a = \exp \left[ -4\pi Z \sqrt{0.0993 \text{MeV}} \sqrt{E_a (\text{MeV})} + \beta \frac{ZR_{nuc}(m)}{7.3 \times 10^{-7}} \right] \]

\[ F_{\text{coul}} = k_c q_i q_j \frac{r_i}{r^2} \quad F_{\text{radiat}} = \frac{mv^2}{r} \quad E_n = -\frac{2Ze^2}{\delta_0^2 a_0 n^2} = -\frac{13.6Z^2}{n^2} \text{ (eV)} \quad r_n = \frac{4\pi z^2 \hbar^2 n^2}{m_e Z^2 e^2 n^2} = a_0 \frac{n^2}{Z} \]

\[ a_0 = \frac{4\pi z^2 \hbar^2}{m_e e^2} = 0.529 \text{ Å} \quad v_n = \frac{n \mu_n}{m_r} = \frac{1}{\lambda} \quad Z^2 R^2 \left[ \frac{1}{n^2} - 1 \right] \quad \mu_n = m_e \left[ \frac{M}{M + m} \right] \]

\[ x = r \sin \theta \cos \phi \quad y = r \sin \theta \sin \phi \quad z = r \cos \theta \quad dV = r^2 dr \sin \theta d\phi d\phi \]

\[ \psi_{n,\alpha}(r, \theta, \phi) = R_n(r) \Theta_{\alpha}(\theta) \Phi_{\alpha}(\phi) \]

\[ P_n(r) = r^2 R_n^2(r) \]

\[ I \propto (|\psi_{\text{final}}|/|\psi_{\text{initial}}|)^2 \quad \Delta \epsilon = \pm 1, \Delta m_r = 0, \pm 1 \]

\[ \mu = iA \quad E = -\mu B \quad \bar{\mu}_t = -\frac{e}{2m} \quad \bar{\mu}_t = -\frac{e}{m} \quad \bar{s} = -2\mu B \quad \bar{s} \]

\[ \mu_B = \frac{e\hbar}{2m_e} = 9.274 \times 10^{-24} \text{ J/Tesla} \quad \mu_B = \frac{e\hbar}{2m_p} = 5.058 \times 10^{-27} \text{ J/Tesla} \]

\[ \psi_j^{\text{MO}}(\vec{r}) = \sum_{\text{Atomic A Orbitals} i} c_{i,j} \psi_{Al}(\vec{r}) \quad \Psi_K(x) = u_K(x) e^{ikx}, \text{ where } u_K(x) = u(x + A) \]

\[ V = \sum_{i=1}^{\infty} \left( \frac{\pm \text{ or } -\text{n} q_i e^2}{4\pi \varepsilon_0 f_i} \right) + \lambda e^{\frac{r_i}{\lambda}} \]

\[ F_{MB} = A \exp(-E/k_B T) \quad F_{BE}(E) = \frac{1}{B_2 \exp(E/k_B T) - 1} \quad F_{FD}(E) = \frac{1}{\exp[(E - F)/k_B T] + 1} \]

\[ \lambda_{max} = 2.898 \times 10^{-3} \text{ m-K} \quad l(\lambda, T) = \frac{2\pi c^2 \hbar}{\lambda^5} \cdot \frac{1}{e^{hcT/k_B T} - 1} \quad R(T) = e^\sigma T^4 \]

\[ E_F = \frac{k^2}{2m_e} \quad g(E) = \left(\frac{2m_e}{2\pi^2 \hbar^2}\right)^{1/2} \quad k_F = \left(3m^2 n_{val} \right)^{1/3} \]

\[ B(\frac{1}{2} X) = [N m_n + Z m(H) - M(\frac{1}{2} X)] c^2 = a_n A - a_A A^{2/3} - \frac{3(Z - 1)e^2}{5} - a_s (N - Z)^2 + \delta \]

\[ R_{nuc} \approx (1.2 \times 10^{-15} m) A^{1/3} \quad Q = [M_{\text{initial}} - M_{\text{final}}] c^2 \]

---Tear off this sheet and begin exam---
Physics 9HE-Modern Physics
Final Examination
March 22, 2012
(100 points total)

Name (printed)____Suggested answers_____________________
Name (signature)_____________________________________
Student ID No._____________________________________

--------------------------------------------------------------------------

[1] (25 points) Answer the following questions with brief statements or calculations.

(a) State three experimentally verified consequences of General Relativity:
Any three of these:
Precession of the perihelion of Mercury
Gravitational lensing
Black holes
Time dilation or contraction for clocks in different parts of a gravitational field
Frame dragging

(b) The so-called “L x-rays” are emitted from a copper atom in which an initial 2p vacancy is created. What transitions from the n = 3 shell are permitted in generating these x-rays and why?

Dipole selection rules are: \( \Delta \ell = \pm 1 \) and \( \Delta m_\ell = 0, \pm 1 \), so the allowed transitions are from 3d with \( \Delta \ell = 1-2 = -1 \) and 3s with \( \Delta \ell = 1-0 = +1 \), and this would get full credit. In more detail using the other selection rule, we would have:

- For 3d: \( \ell = 2, m_\ell = -2, -1, 0, +1, +2 \)
- For 3s: \( \ell = 0, m_\ell = 0 \)
- For 2p: \( \ell = 1, m_\ell = -1, 0, +1 \)

(c) A red laser is used to produce a perfect sinusoidal traveling light wave of frequency \( \nu_0 = 10^{14} \text{ Hz} \). However, by special means, the wave is abruptly cut off at each end so that it has a finite length in time
of $10^{-13}$ s. If this finite wave is now described in a Fourier representation, what would be the approximate range of frequencies $\Delta \nu$ involved?

(d) Define degeneracy in quantum mechanics and give one example of it from the systems we have studied.

(e) The position of an electron along the x direction is measured with an uncertainty of 1 Å. What can you say about the uncertainty in its x momentum? What can you say about the uncertainty in its y momentum?

\[ \Delta x \Delta p_x \geq \frac{h}{4\pi} \]
\[ \Delta p_x \geq \frac{h}{4\pi \Delta x} = \frac{6.63 \times 10^{-34}}{1 \times 10^{-10}} = 6.63 \times 10^{-24} \text{ kg m/s} \]
\[ \Delta p_y \text{ is not involved with x coordinate measurement, so cannot say anything} \]

(f) In the laser, what condition on level populations is required for successful operation? Explain briefly with a level diagram.

There must be some kind of “population inversion”, such that an excited state is much more populated than some lower-lying state. Schematically, this looks like:

\[ \text{Somehow} \]
\[ N_2 \text{ (High population)} \]
\[ \text{Somehow} \]
\[ N_1 \text{ (Low population)} \]

\[ \text{NEDO A POPULATION INVERSION, AS SHOWN AT LEFT, IN ORDER FOR STIMULATED \ EX CITE TO DOMINATE} \]
(g) A certain real solid is known to have an electronic band structure that is very free-electron like. Sketch on the diagram below what the lowest band would look like, indicating the specific equation for it:

[2] (10 points) I am being followed by the CHP on I-80, and, by using Doppler radar emitted at 5 GHz from his car traveling behind me at 70 mph, the officer asserts that I was traveling at 95 mph and prepares to write me a ticket. (1 mile = 1.601 km)

(a) What would be the frequency shift in the radiation in Hz reflected back from my vehicle, as seen by the officer’s radar?

95 mph = 42.2 m/s, 70 mph = 31.1 m/s. Relative velocity is thus 11.1 m/s, and incoming frequency to my car is:

\[ v' = v_0(1-v/c) = 5 \times 10^9 \left( 1 - \frac{11.1}{(3 \times 10^8)} \right) \]

The frequency reflected back to CHP is then reduced by the same amount, or

\[ v'' = v'(1-v/c) = 5 \times 10^9 \left( 0.999999963 \right) ^2 \]

and the difference between the starting and ending frequencies would be:

\[ v_0 - v'' = 5 \times 10^9 \left[ 1 - (0.999999963)^2 \right] = 370 \text{ Hz} \]

(b) But I have now upgraded my vehicle and can fly above ground with it. Would it help any to elevate myself above ground to make the returning reflected radiation seem to show no difference in speed between our two vehicles, thus avoiding a speeding ticket (even if I get one for flying without a pilot’s license)? You may assume that gravity is constant over this elevation, and neglect time-dilation effects, but think carefully about what happens to the outgoing and returning radiation.

If you don’t think so carefully, you could just use→
\[ \Delta \nu = \frac{\nu_0 - \nu''}{\nu_0} \approx -\frac{gH}{c^2} = \left[ 1 - (0.999999963)^2 \right] = 7.4 \times 10^{-8} \]

so

\[ H = 7.4 \times 10^{-8} c^2 / g = 6.78 \times 10^8 \text{ m} \]

However, this is not a good way to beat a speeding ticket, and in fact, it wouldn't even work in principle, because the outgoing wave would lose energy and decrease frequency, while the return wave would gain the same amount and increase frequency. So the effects would cancel, and I've spent a lot of money on making my car fly for nothing! [3/4 credit for missing the "however" and just doing the correct calculation for the outgoing radiation, and full credit for just stating the last paragraph correctly.]

[3]) (10 Points) We have considered in lecture and laboratory the triple-well potential, and some of Randy's wave functions for the lowest three states are shown below:

As a more realistic example of this type of potential, consider the molecule C\textsubscript{3}H\textsubscript{4}, with three C atoms in a line, and for which a few of its molecular electronic wave function probability distributions are shown below (but not in energy order):

\begin{enumerate}
  \item[(A)]
  \item[(B)]
  \item[(C)]
\end{enumerate}

The blue and green contours represent opposite signs for the \( \Psi \) giving that probability distribution.

(a) Neglecting the influence of the hydrogens, indicate which of the wave functions A, B, or C for this molecule are connected in general origin with which of the wave functions 1, 2, or 3 of the triple-well potential, including your estimate of their order in energy.

They must have the same order in energy and also be of even-odd-even parity, since the potentials of the three C atoms (neglecting the effect of H) are even along the molecular axis. So the correlation is:

C is related to 1, A is related to 2, and B is related to 3, and they are in that order of energy: \( E_C < E_A < E_B \).

(b) Again neglecting the hydrogens, Indicate the bonding character and the approximate atomic-orbital makeup of the wave function denoted by (A), using either a sketch or a clearly labeled equation. Let \( z \) be along the internuclear axis of the molecule.
This is a bonding wave function (of \( \sigma \) character, but this was not needed for a full credit answer) and anything like the equations or drawings below OK:

\[
\psi_A \approx C_{1-2s} - C_{2-2pz} + C_{3-2s} \quad \psi_A \approx C_{1-2pz} - C_{2-2pz} + C_{3-2pz}
\]

\[\begin{align*}
&\text{[4] (15 points)} \quad \text{A particle of energy } E \text{ approaches a barrier of height } V_0 \text{ and width } L \text{ and } E \text{ is much greater than } V_0. \text{ Take the value of } k \text{ for the particle in the region of the barrier to be some number } k_0.

(a) Indicate the form of the time-independent wave function in each or regions 1, 2, and 3, and the boundary conditions that would have to be satisfied to solve this problem.

Forms of wave functions indicated in drawing above.

Boundary conditions on continuity of \( \Psi \) and \( d\Psi/dx \) are:

\[x = 0 : A + B = C + D \quad \text{and } ikA - ikB = i k_0 C - i k_0 D \rightarrow k(A - B) = k_0(C - D) \]

\[x = L : Ce^{ikL} + De^{-ikL} = Ee^{ikL} \quad \text{and } i k_0 Ce^{ikL} - i k_0 De^{-ikL} = ikEe^{ikL}\]

(b) Show that the reflection coefficient \( R \) in this high-energy limit is given approximately by

\[\left\{ \frac{V_0 \sin(k_0L)}{2E} \right\}^2\]

and thus still shows quantum effects. Note that \( (1 + x)^{-1} \approx 1 - x \text{ for small } x \).

\[T = \left[ 1 + \frac{V_0^2 \sin^2(k_0L)}{4E(E - V_0)} \right] \approx \left[ 1 + \frac{V_0 \sin^2(k_0L)}{4E(E - V_0)} \right] \approx \left[ 1 - \frac{V_0^2 \sin^2(k_0L)}{4E^2(1 - V_0 / E)} \right]
\]

But \( T+R = 1, \) so

\[R = \frac{V_0^2 \sin^2(k_0L)}{4E^2} - \left[ \frac{V_0 \sin^2(k_0L)}{4E} \right]^2\]

(c) What is the condition on the wavelength of the particle for which \( R \) is a maximum? Illustrate two non-trivial cases with a sketch on the diagram above.

The condition for a maximum reflectivity are for \( \sin(k_0L) = 1 \), or \( k_0L = (2\pi/\lambda_0)L = \pi/2, 5\pi/2, 9\pi/2, 13\pi/2,... = \pi/2 + n(2\pi) \), as shown below:
with \( n = 0, 1, 2, \ldots \) or \( 2L/\lambda_0 = \frac{1}{2}, \frac{5}{2}, \frac{9}{2}, \frac{13}{2}, \ldots = \frac{1}{4} + n \), in which case the “roundtrip” distance of the two components inside the potential step region \( Ce^{ikx} + De^{-ikx} \) makes them just out of phase at the reflecting boundary at \( x = 0 \) with the reflected wave represented by \( Be^{-ikx} \), thus minimizing transmission. Two cases for \( n = 1 \) and \( n = 2 \) like this are shown above.

[5] (25 points)

One of the wavefunctions for the hydrogen atom has the form:

\[
\psi_{211}(r, \theta, \phi) = C \left( \frac{r}{a_0} \right) e^{-\frac{r}{a_0}} \sin \theta e^{i\phi}, \text{ with } C = \text{some constant.}
\]

(a) What is the expectation value for the square of angular momentum in this state? Do this one the easiest way you know how; think eigenfunction!

Just use the known eigenfunction relationship for the H-atom wavefunctions (you need to remember this), which gives for the \( n = 2, \ell = 1, m = 1 \) case shown above: \( \langle \ell^2 \rangle = \hbar^2 \ell (\ell + 1) = 2 \hbar^2 \).

(b) What is the time-dependent wavefunction for this state? Specify the energy precisely.

Just add the energy exponential, to give

\[
\psi_{211}(r, \theta, \phi, t) = \psi_{211}(r, \theta, \phi)e^{-\frac{iE}{2\hbar}t}, \text{ with } E_2 = -\frac{13.6(1)^2}{2^2} = 3.4 \text{ eV}
\]

(c) How would you determine the average radius of an electron in this state? Set up the integral involved, but you need not evaluate it.

\[
\int_{0}^{\infty} \int_{0}^{\pi} \int_{0}^{2\pi} \psi_{211}^* (r, \theta, \phi) r^2 \sin \theta d\theta d\phi = \left( \frac{C}{a_0} \right)^2 \frac{2\pi}{a_0^3} e^{-\frac{r}{a_0}} \sin^2 \theta e^{i\phi} r^2 dr \sin \theta d\theta d\phi
\]

with integration over \( r(0 \text{ to } \infty) \), \( \theta(0 \text{ to } \pi) \), and \( \phi(0 \text{ to } 2\pi) \), so this gives finally

\[
= \left( \frac{C}{a_0} \right)^2 2\pi \int_{0}^{\infty} e^{-\frac{r}{a_0}} r^2 dr \sin^2 \theta d\theta
\]

(d) Qualitatively sketch the radial probability density for this state, including an indication of the precise positions of any nodes.

The radial probability density is just

\[
\text{sink}_0 L
\]

\[
\pi/2 \quad \pi/2+2\pi \quad \pi/2+4\pi \quad \pi/2+6\pi \quad \pi/2+8\pi
\]

\[
k_0 L
\]
\[ P_{21}(r) = r^2 R_{21}^0(r) = C \left( \frac{r}{a_0} \right)^2 e^{-\frac{r}{a_0}} r^4 = \left( \frac{C}{a_0} \right)^2 e^{-\frac{r}{a_0}} r^4 \]

and, not needed for complete credit, but this will have a maximum at

\[ \frac{dP_{21}(r)}{dr} = 0 \text{ or } -\frac{1}{a_0} e^{-\frac{r}{a_0}} r_{\text{max}}^4 + 4 e^{-\frac{r}{a_0}} r_{\text{max}}^2 = 0, \]

which gives

\[ r_{\text{max}} = 4a_0 \]

which will look something like

and is also shown in one of the lecture slides on the next page:

\[ \Phi_{n,l,m} = R_l(r) e^{im\phi} \]

(e) The operator for the z component of angular momentum can be shown to be given by \( \hat{L}_z = -i\hbar \frac{\partial}{\partial \phi} \). Show that this wavefunction is an eigenfunction of this operator, and state its eigenvalue.

Operator only acts on the phi-dependent part of the wave function, so we have

\[ \hat{L}_z e^{im\phi} = -i\hbar \frac{\partial}{\partial \phi} e^{im\phi} = -i\hbar(i)e^{im\phi} = \hbar \]

as expected since \( m_z = 1 \)

(f) If this wave function was now modified in form so as to represent one of the electrons in an Ar atom with a configuration of \( 1s^22s^22p^63s^23p^6 \), to what approximate screened nuclear charge would it correspond?

Nuclear charge is here 18, and minimally the 2 1s electrons will be “inside of” the 2p electrons, so 18-2 = 16 is probably the best answer. But the 2s electrons and to some degree also the other 2p electrons, will screen the nuclear charge of a given 2p electron, so 18-2-2 =14, or even 18-2-2-5 would be adequate answers here.
[6] (15 points) Consider the nuclide $^{49}_{24}Cr_x$ of chromium, with atomic mass of 48.951341 u.

(a) What is x here, and what is the binding energy of this nuclide per nucleon?

\[
x = 49 - 24 = 25
\]

\[
E_b_{\text{real nucleon}} = \frac{E_b}{49}\left[24\, m_p c^2 + 25\, m_n c^2 - M \left(\frac{m_e}{m}\right)^2 c^2\right] = \frac{1}{49}\left[24\, (938.27) + 25\, (939.57) - 48.951341\, (931.49)\right] = \frac{1}{49}\left[22123.2 + 23499.3 - 45597.7\right] = \frac{446.1}{49} = 8.36\, \text{MeV/nucleon}
\]

\[
= 13.3(1) \times 10^{-13}\, J = 1.33(1) \times 10^{-12}\, J
\]

(b) This nuclide decays by positron emission to form a nuclide of vanadium = $^{??}_{??}V$. Write down the overall reaction, including the new nuclide that would be formed.

Positron emission reduces the atomic no. Z by one, and overall reaction is:

\[
^{49}_{24}Cr_{25} \rightarrow ^{49}_{23}V_{26} + e^+ + \nu_e
\]

(c) Which one of the four fundamental interactions is responsible for the force between the quarks inside the nucleons of this nuclide, and what is the particle mediating this force?

The strong interaction is responsible for quark interaction and the mediating particle is the gluon.

(d) How many total quarks reside inside the nuclide $^{49}_{24}Cr_x$?

Each nucleon contains three quarks, so the number of quarks is just $3(24+25) = 147$

--End of examination--