Physics 9HE-Modern Physics
Final Examination
March 19, 2011
(100 points total)
You may tear off this sheet.

Miscellaneous data and equations:
\[ c = 3.00 \times 10^8 \text{ m/s} \quad e = 1.60 \times 10^{-19} \text{ C} \quad 1 \text{ eV} = 1.60 \times 10^{-19} \text{ J} \quad 1 \text{ Å} = 10^{-10} \text{ m} \]
\[ M_{\text{Sun}} = 2 \times 10^{30} \text{ kg} \quad M_{\text{Earth}} = 5.98 \times 10^{24} \text{ kg} \quad r_{\text{Earth}} = 6.38 \times 10^6 \text{ m} \]
\[ m_e = 9.1094 \times 10^{-31} \text{ kg} = 0.5110 \text{ MeV/c}^2 \quad m_p = 1.6726 \times 10^{-27} \text{ kg} = 938.27 \text{ MeV/c}^2 \]
\[ m_n = 1.6749 \times 10^{-27} \text{ kg} = 939.57 \text{ MeV/c}^2 \]
\[ m(1\text{H}) = 1.0078 \text{ u} \quad G = 6.67 \times 10^{-11} \text{ Nt-m}^2/\text{kg}^2 \quad g = 9.81 \text{ m/s}^2 \]
\[ \hbar = 6.63 \times 10^{-34} \text{ J-s} \quad \gamma = 5.67 \times 10^{-8} \text{ W-m}^{-2}\text{K}^{-4} \]
\[ n = h/2 \pi \quad \pi = 3.1415 \]
\[ k_B = 1.38 \times 10^{-23} \text{ J-K}^{-1} \quad a_0 = 0.529 \text{ Å} \]

**Einstein's Mass-Energy Equivalence:**
\[ E^2 = m^2c^4 + m^2c^2 \]
\[ E = \gamma mc^2 \quad p = \gamma m \]
\[ E = \gamma mc^2 \quad r_{\text{Earth}} = 6.38 \times 10^6 \text{ m} \]

**Energy Distribution:**
\[ n(\lambda) = N_0 \exp(-\lambda \tau_0) = N_0 \exp(-0.693 \lambda \tau_{1/2}) \]
\[ k_C \equiv \frac{1}{4 \pi \varepsilon_0} = 8.98 \times 10^9 \text{ N-m}^2/\text{C}^2 \quad \lambda_{\text{max}}T = 2.898 \times 10^{-3} \text{ m-K} \]
\[ I(\lambda, T) = \frac{2\pi^2\hbar}{\lambda^5} \frac{1}{e^{\frac{\varepsilon_0}{T}} - 1} \]

**Wave Function:**
\[ \psi(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{2\pi n}{\ell} x\right) + \sum_{n=1}^{\infty} b_n \sin\left(\frac{2\pi n}{\ell} x\right), \quad \text{with } k_n = \frac{2\pi n}{\ell}, \text{ and} \]
\[ a_n = \text{aver. of } \psi \text{ over } \ell, \quad a_n = \frac{2}{\ell} \int_0^\ell \psi(x) \cos\left(\frac{2\pi n}{\ell} x\right) dx; \quad b_n = \frac{2}{\ell} \int_0^\ell \psi(x) \sin\left(\frac{2\pi n}{\ell} x\right) dx \]
\[ \psi(x) = \frac{1}{\sqrt{2\pi}} \int c(k) e^{ikx} dk, \text{ with } c(k) = \frac{1}{\sqrt{2\pi}} \int \psi(x) e^{-ikx} dx \]
\[ \hat{\mathbf{H}} \psi = i\hbar \frac{\partial \mathbf{\psi}}{\partial t} \quad \hat{H} = \hat{\mathbf{K}} + V \quad \mathbf{\psi} = \psi e^{iEt/\hbar} = \psi e^{-i\lambda t} \quad \hat{\mathbf{H}} \psi = E\psi \]
\[ \hat{\mathbf{p}}_x = -i\hbar \frac{\partial}{\partial x} \quad \hat{\mathbf{K}}_x = -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} \quad \Delta A = \sqrt{\langle A^2 \rangle - \langle A \rangle^2} < A >= \int \psi^* \hat{\mathbf{A}} \psi dx \quad \hat{\mathbf{A}} \psi_a = a \psi_a \]
\[ k = \sqrt{2m(E-V)/\hbar^2} \quad \psi \propto e^{\pm i k \xi} \quad \psi \propto \sin kx, \cos kx \quad \kappa = \alpha = \frac{1}{\delta} = \sqrt{2m(V-E)/\hbar^2} \quad \psi \propto e^{\pm i k \xi} \]

\[ \psi_n = \left( \frac{2}{L} \right)^{1/2} \sin \left( \frac{n\pi x}{L} \right) \quad E_n = \frac{n^2\hbar^2}{2mL^2} \]

\[ \psi_n = H_n(x)e^{-\alpha x^2/2} \quad \alpha = \sqrt{\frac{m\kappa}{\hbar^2}} \quad \omega = \sqrt{\frac{\kappa}{m}} \quad E_n = \left( n + \frac{1}{2} \right)\hbar \omega \]

\[ \psi_J = Y_{m1}(\theta, \phi) = \Theta_{m1}(\theta)\Phi_{m1}(\phi) \quad E_J = \frac{\hbar^2 J(J+1)}{2I} \quad l = m,m_1+R_{12} \]

\[ T = \left[ 1 + \frac{V_0^2 \sin^2(kL)}{4E(E-V_0)} \right]^{-1/2} \quad T = \left[ 1 + \frac{V_0^2 \sin^2(kL)}{4E(E-V_0)} \right]^{-1/2} \approx 16 \left( \frac{E_0 - E}{V_0} \right) e^{-2kL} \quad \text{(when } kL \gg 1) \]

\[ T_{FE} \approx \exp \left[ -\frac{4\beta^2}{3e\hbar} \frac{dV}{dx} \right] \quad I_{STM} \propto e^{-2kL} \quad \lambda_a = f_{cor}T_a; \quad T_a = \exp \left[ -4\pi \frac{0.0993 \text{ MeV}}{E_a(\text{MeV})} + 8 \frac{ZR_{\text{nuc}}(m)}{7.3 \times 10^{-7}} \right] \]

\[ F_{\text{coul}} = k_C \frac{q_i q_2}{r^2} \quad F_{\text{radial}} = \frac{mv^2}{r} \quad E_n = -\frac{Z^2e^2}{8\pi\epsilon_0\alpha_n}n^2 = -\frac{13.6Z^2}{n^2} \text{(eV)} \quad r_n = \frac{4\pi\epsilon_0\hbar^2}{m_e} \text{n}^2 = a_0 \frac{n^2}{Z} \]

\[ a_0 = \frac{4\pi\epsilon_0\hbar^2}{m_e} = 0.529 \text{ Å} \quad v_n = \frac{n\hbar}{m_r r_n} \quad \lambda = \frac{1}{\lambda} = Z^2R_n \left[ \frac{1}{n_1^2} - \frac{1}{n_2^2} \right] \quad \mu_e = m_e \frac{M}{M+m_0} \]

\[ x = r \sin \theta \cos \phi \quad y = r \sin \theta \sin \phi \quad z = r \cos \theta \quad dV = r^2 dr \sin \theta d\phi d\phi \]

\[ \psi_{ncm}(r, \theta, \phi) = R_m(r)\Theta_{m1}(\theta)\Phi_{m1}(\phi) = R_{nc}(r)Y_{m1}(\theta, \phi) \quad P_{nc}(r) = r^2R_{nc}(r) \]

\[ l \propto \left| \psi_{\text{final}} \right| f_{\text{inital}} \right| \quad \Delta l = \pm 1, \Delta m_r = 0, \pm 1 \]

\[ \mu = iA \quad E = -\mu + \vec{B} \quad \vec{\mu} = -\frac{e}{2m} \vec{L} = -\mu_0 \frac{\vec{L}}{\hbar} \quad \vec{\mu}_s = -\frac{e}{m} \vec{S} = -2\mu_0 \frac{\vec{S}}{\hbar} \]

\[ \mu_B = \frac{e\hbar}{2m_e} = 9.27 \times 10^{-24} \text{ J/Tesla} \quad \mu_N = \frac{e\hbar}{2m_p} = 5.058 \times 10^{-27} \text{ J/Tesla} \]

\[ \psi_{m0}(\vec{r}) = \sum_{\text{Atoms Orbitals } i} c_{Alj} \phi_{Ai}(\vec{r}) \quad \psi_{K}(x) = u_K(x)e^{iKx}, \text{ where } u_K(x) = u(x+A) \]

\[ V = \sum_{l=1}^{n} \left( + or - \right)n_l q_l e^2 / 4\pi\epsilon_0 r_l + \lambda e^2 / r_l \]

\[ F_{MB} = A \exp(-E/k_B T) \quad F_{BE}(E) = \frac{1}{B_8 \exp(E/k_B T) - 1} \quad F_{FD}(E) = \frac{1}{\exp[(E-E_F)/k_B T] + 1} \]

\[ \lambda_{\text{max}} T = 2.898 \times 10^{-3} \text{ m-K} \quad I_l(T) = \frac{2\pi^2\hbar^5}{\lambda^5 L} \cdot \frac{1}{\hbar c e^4 \kappa T} - 1 \quad R(T) = e\sigma T^4 \]

\[ E_F = \frac{\hbar^2 k_F^2}{2m_e} \quad g(E) = \frac{(2m_e)^{3/2}}{2\pi^2 \hbar^3} E^{1/2} \quad k_F = (3\pi^2 n_{\text{val}})^{1/3} \]

\[ B(\frac{1}{2}X) = \frac{Zm_n}{Zm_1} \left( M(\frac{1}{2}X) - M(\frac{1}{2}X) \right)^2 \approx a_0 A - a_0 A^{1/3} - \frac{3}{5} Z(Z-1)e^2/a - a_0 (N-Z)^2/A + \delta \]

\[ R_{\text{nuc}} \approx (1.2 \times 10^{-15} m) A^{1/3} \quad Q = [M_{\text{initial}} - M_{\text{final}}] c^2 \]

---Tear off this sheet and begin exam---
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Name (printed)_______________________________________
Name (signature)_______________________________________
Student ID No._______________________________________

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[1] (66 points) Answer the following independent questions briefly:

(a) (5 points) State the fundamental postulate of General Relativity.

*Principle of Equivalence: No experiment can be done in a confined space that can detect the difference between a uniform gravitational field and an uniform acceleration. Or any nearly equivalent statement. (Partial credit for answering with one of the observable verification of this.)*

(b) (7 points) The Space Station travels at an altitude of about 370 km and at a speed of 28,000 km/hour and has a clock inside. Assume that the gravitational force is essentially constant from the ground to this altitude and calculate the fractional changes in this clock’s time as observed on the ground, including both Special and General Relativity, and specify the sign of each change (faster, slower than a ground based clock).

\[
\Delta T = \frac{9H}{c^2} = \frac{9 \times 10^5}{3 \times 10^8} = 3 \times 10^{-11} \text{ (faster)}
\]

\[\text{Special Relativity: Stationary observers measure longer periods of time than moving observers, i.e., moving clocks run slow.}\]

Then, \( T = T_0 \Rightarrow \Delta T = \frac{T - T_0}{T_0} \)

\[
\Delta T = \frac{T - T_0}{T_0} = 1 - \sqrt{1 - \frac{v^2}{c^2}} \\
= \frac{1 - \left(\frac{28,000 \text{ km}}{10 \text{ km}} \times \frac{1 \text{ hr}}{3600 \text{ s}}\right) \left(\frac{1 \text{ km}}{1000 \text{ m}} \times \frac{1 \text{ hr}}{3600 \text{ s}}\right)^2}{1 - \frac{v^2}{c^2}} \\
= 3.4 \times 10^{-10} \text{ (slower)}
\]
(c) (6 points) Sketch the energy level system for a semiconductor which has been p doped, labelling the various electronic states and bands, the position of the Fermi level, and indicating the likely bonding and anti-bonding character of the bands involved as appropriate.

(d) (6 points) (Name the four fundamental forces and the particles which mediate them, and indicate what they are responsible for in nature (i.e. give an example of what they cause).

(e) (9 points) (Two parts) Consider the rotational excitations of a molecule of HCl, with atomic masses of $M_H = 1.0078$ u and $M_{Cl} = 34.9688$ u and a bond length of 1.26 Å.

1. (4 points) What frequency of light would be necessary to excite this molecule from its ground state wave function $\Psi_{00}(\theta, \phi)=\frac{1}{2}\sqrt{\frac{1}{\pi}}$ to a first excited state wave function $\Psi_{1,1}(\theta, \phi)=-\frac{1}{2}\sqrt{\frac{3}{2\pi}}\sin\theta\exp(i\phi)$?
(2) (5 points) Consider now light with polarization along the \( x = r \sin \theta \cos \phi \) direction and write down the integrals that would have to be evaluated in order to determine whether this transition from the ground state to the first excited states would be allowed.

\[
\langle \psi_{11} | x | \psi_{00} \rangle = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_{0}^{\pi} \int_{0}^{2\pi} \psi_{11}^* \psi_{00} \sin \theta \cos \phi \, \sin \theta \cos \phi \, \sin \theta \cos \phi \, \sin \theta \cos \phi \, \sin^3 \theta \, d\theta = 0
\]

\[
\cos \phi
\]

(f) (6 points) Write down all the eigenfunction relationships satisfied by the hydrogenic wave function \( \psi_{321}(r, \theta, \phi) \) and give the value of each eigenvalues in terms of fundamental constants. (Full credit for energy plus any two of the others.)

\[
\hat{H} \psi_{321} = E_3 \psi_{321}, \text{ where } E_3 = -(13.6 \text{ eV}) \left( \frac{2^2}{3} \right) = -1.51 \, 2^2 \text{ eV}
\]

\[
\hat{L}_z \psi_{321} = L_z \psi_{321}, \text{ where } L_z = L(1) = L(1) = L(1)
\]

\[
\psi_{321} = (-1)^2 \psi_{321} = (+1) \psi_{321}, \text{ even parity}
\]

(g) (5 points) Would the following be a suitable approximate wavefunction for two neutrons which overlap in space, where \( a \) and \( b \) represent two different sets of quantum numbers including the neutron spin and \( C \) is a suitable normalization constant?

\[
\psi(r_1, \vec{s}_1; r_2, \vec{s}_2) = C \left[ \Phi_a(r_1, \vec{s}_1) \Phi_b(r_2, \vec{s}_2) + \Phi_b(r_1, \vec{s}_1) \Phi_a(r_2, \vec{s}_2) \right]
\]

Explain why or why not.

\[
\text{WHERE FOR ANTI-SYMMETRY REQUIRED OF } \psi, \text{ SINCE NEUTRON = FERMION.}
\]

\[
\hat{P}_{12} \psi(r_1, \vec{m}_{s_1}; r_2, \vec{m}_{s_2}) = \hat{P}_{12} \left[ \Phi_a(r_1, \vec{m}_{s_1}) \Phi_b(r_2, \vec{m}_{s_2}) + \Phi_b(r_1, \vec{m}_{s_1}) \Phi_a(r_2, \vec{m}_{s_2}) \right]
\]

\[
= \left[ \Phi_a(r_1, \vec{m}_{s_1}) \Phi_b(r_2, \vec{m}_{s_2}) \Phi_a(r_1, \vec{m}_{s_1}) \right]
\]

\[
\Rightarrow \text{ SYMMETRIC, SO NOT OK FOR NEUTRONS}
\]
(h) (10 points) (Two parts) A million particles of the same energy $E$ hit a rectangular potential barrier of height $V_0 = 16E$ and width $L$ per second. It is observed that only 1,000 particles emerge on the other side of the barrier per second.

1,000,000 $\leftarrow$ $V = 0$ $\rightarrow$ 1,000

(1) How many particles will be reflected back from the barrier?

*Simple, as particles are not being destroyed, and $R+T=1$: So Reflected = 1,000,000 - 1,000 = 999,000.*

(2) If the thickness of the barrier were doubled, use a simple limiting formula to estimate how many particles would now emerge per second for the same incident flux?

\[
T(E) = \frac{E^4}{V_0^4} \left(1 - \frac{E}{V_0}\right)e^{-2KL} = \frac{10^3}{16} = 10^{-3}
\]

\[
\Rightarrow \frac{T_{2L}(E)}{T_L(E)} = \frac{e^{-4XL}}{e^{-2KL}} \approx 10^{-3} \Rightarrow T_{2L}(E) \approx 10^{-6}
\]
(i) (6 points) Show that a one-dimensional wave function for an electron in a periodic solid whose atoms repeat over a distance $A$ that is of the Bloch form:

$$\Psi_{k}(x) = u_k(x)e^{ikx}, \text{where } u_k(x) = u(x + A)$$

yields a probability density that is periodic in $A$.

Just calculate the probability density, as:

$$\Psi_k^*(x)\Psi_k(x) = u_k^*(x)e^{-ikx}u_k(x)e^{ikx} = u_k^*(x)u_k(x)$$

$$= |u_k(x)|^2 = |u_k(x + A)|^2 = |u_k(x + 2A)|^2 = \ldots = |u_k(x + NA)|^2$$

Thus, it is periodic on the lattice of atoms.

(j) (6 points) The $\Sigma^+$ particle is composed of two up quarks "u" (each with mass 360 MeV/c$^2$ and charge $2e/3$) and a strange quark "s" (with mass 540 MeV/c$^2$ and charge $-e/3$). Estimate the $\Sigma^+$ mass (assuming weak binding) and calculate its charge.

$$M_{\Sigma^+} \approx 2M_{u} + M_{s} \approx 2(360) + 540 = 1,200 \text{ MeV/c}^2$$

If weakly bound, then

$$M_{\Sigma^+} \approx 1,189.37 \pm 0.07$$

In fact, the $\Sigma^+$ has a mass of 1,189.37 ± 0.07, so we are not too far off, and it seems weakly bound.

[2] (18 Points)
Consider a particle trapped inside the one-dimensional vee-shaped potential well shown below, whose form over the full range of $x$ is $V(x) = A|x|$, where $A$ is a constant.

(a) (6 points) Qualitatively sketch on the diagram above the form of the wavefunctions for the ground state at $E_1$ and the first two excited states at $E_2$ and $E_3$ over the full range of $x$, being careful to show the relative wavelengths and amplitudes in each region.

(b) (6 points) What would be the expectation value of momentum for any of the states above? Indicate the integral involved, but use symmetry to expedite your reasoning.
(c) (6 points) Consider now the Correspondence Limit for this problem for $E_n$ with $n$ very large, and sketch a typical probability distribution in this limit on the diagram below, being as quantitative as you can with the relative amplitudes as a function of $x$.

[3] (16 points)

We are hearing much these days about the problems with the nuclear reactors in Japan, and in particular the problems associated with $^{131}_53X$, an isotope of iodine, which is selectively incorporated into the thyroid gland, possibly causing cancer.

(a) (3 points) What is $X$ above? $X = 131 - 53 = 78$
(b) (5 points) What is the binding energy per nucleon of $^{131}_{53}X$ if its total mass is 130.9061 u?

Binding energy per nucleon = \[
\frac{1}{131} [53m_{^{131}_{53}X} + \frac{1}{2}78m_{^{128}_{78}X} - m_{^{131}_{53}X}] \]

= \[
\frac{1}{131} [53 \cdot 0.0078 \times 931.494 \text{ MeV}/c^2 + 78 \cdot 39.57 \text{ MeV}/c^2 - 130.9061 \cdot 31.494 \text{ MeV}/c^2 ] \]

= \[
\frac{1}{131} [49754.26 + 73286.46 - 121938.25] \text{MeV} = 1102.47 \text{MeV}/\text{nucleon} \]

Which agrees in magnitude with the values given in Thornton and Rex, Fig. 12.6.

(c) (4 points) $^{131}_{53}X$ decays via $\beta^-$ with a half-life of 8.02 days, with the $\beta^-$ exposure being cancer producing. Write down the decay scheme for this process, including all particles involved, assuming that the chemical symbol for the atom produced is Xe.

$^{131}_{53}X \rightarrow^{131}_{54}Xe + \beta^- + \nu_e$

(d) (4 points) Much of $^{131}_{53}X$ is actually produced as a second step beyond the initial fission processes of $^{236}_{92}U_{144}$, in which an isotope of Tellurium, $^{130}_{52}Te_{78}$, is produced, then absorbs a single neutron, and then decays via $\beta^-$ to $^{131}_{53}X$. Assuming the initial spontaneous fission process produces two neutrons, write down the two overall nuclear reactions (fission and decay), indicating the three relevant numbers for all isotopes:

$^{236}_{92}U_{144} \rightarrow^{130}_{52}Te_{78} +^{106}_{40}Zr_{66} + 2n$

$^{130}_{52}Te_{78} + n \rightarrow^{131}_{52}Te_{79}$

And as an optional final third step:

$^{131}_{52}Te_{79} \rightarrow^{131}_{53}I_{78} + \beta^- + \nu_e$

--End of examination--