Final Exam will be comprehensive over course, with ca. 35% on material up to midterm, and 65% on material after that

**Special Relativity**

- Know the postulates of Special Relativity
- Proper time, proper length
- Time dilation (muon decay, planes circling earth), length contraction
- Radiative decay: as relevant to muon experiment and later discussions of nuclear properties and particle physics
- Doppler shift for light and sound: application to Doppler radar, auto speed monitoring, expansion of universe
- Energy-mass relationships, including reactions in which mass change becomes energy released/taken in (incl. nuclear binding energies and reaction energies from Ch. 12)

**General Relativity**

- Know the postulate of General Relativity (Principle of Equivalence)
- Effect of gravity on light, including shift of frequency, gravitational lensing & dark matter
- Effect of gravity on time (watch confusion with time dilation in Special Relativity)
- Practical examples: GPS satellites for which both Special and General Relativity important; Mössbauer effect with both Doppler and Gen. Rel. present
- Black holes and the Schwartzchild radius
- Motion of the perihelion of Mercury
- Gravity waves, the graviton

**Waves and Introductory Quantum Ideas**

From some basic experiments seen already in 9HC:
- Emission and absorption spectra
- Blackbody radiation:
  - Planck’s model vs. Rayleigh-Jeans model
  - Planck Blackbody formula and its meaning
  - Stefan-Boltzmann formula and Wein displacement laws and their uses
- The photoelectric effect
- Photon energy and photon momentum \(E = pc\) only for particles of zero rest mass like photon..or other particles at very high energy...so be careful!
- X-ray production in an x-ray tube:
  - Via two processes: bremsstrahlung and core electronic transitions
  - Moseley correlations of x-ray energy with atomic number, electron screening, and effective nuclear charges
  - Selection rules in x-ray transitions (from later disc. of time dependence in QM)
- Elastic wavelike scattering of x-rays--Bragg diffraction of x-rays by atomic layers in crystals: \(n\lambda = 2dsin\theta\)
- Electron-positron pair production, the positronium atom, and positron emission tomography (see discussion at end of quarter)
- Emission and absorption spectra of atoms, including selection rules (from later disc. of time dependence in QM)
- De Broglie wavelength for a particle: \(\lambda = h/p\)
- Davisson-Germer diffraction of electrons by crystal surfaces near surfaces:
  - 2d and 3d aspects
- The double-slit experiment with light and electrons: building up images by counting
-Conserving momentum and energy in emission of light and creation/annihilation of particles (e.g. free electron cannot absorb a photon in the photoelectric effect)

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### Basic Concepts and Mathematics of Quantum Mechanics, Including Fourier Transforms

- Complex variables and complex functions—brief review
- Superposition of waves, wave packets, phase and group velocities
- Fourier series and integrals; use of and relationship to the Uncertainty Principles
- Example of RC electrical circuit as a Fourier filter, passing only low frequencies
- Sets of orthogonal, or orthogonal and normalized = orthonormal functions as basis sets for describing any function, including wave functions
- The Heisenberg Uncertainty Principles in position-momentum and energy-time
- Wave packets, group and phase velocities, the Dirac delta function
- Know the postulates of Quantum Mechanics and how to use them, including the general rules for solving and understanding a Q.M. problem (see lecture slides)
- The Correspondence Principle: as energy a/o quantum no. goes up, retrieve classical-like behavior
- Time-dependent and time-independent Schroedinger Equations and wave functions: first example of the use of separation of variables
- Other uses of separation of variables in solving: time-dependent Sch. Eqn., 3D particle in a rigid box, hydrogenic atom, rigid rotor model for molecular rotation, separation of nuclear and electronic motions in molecules, and other q.m. problems
- Calculation of probability densities, expectation values, and dipole matrix elements of operators: make use of even-odd character (parity) and symmetry wherever you can
to simplify this. E.g.- an overall odd integrand, when integrated over -\infty < x < +\infty must be zero; the same is true in 3 dimensions, whether

\[ x^{-\infty} < x < +\infty, y^{-\infty} < y < +\infty, z^{-\infty} < z < +\infty \]  

or

\[ 0 < r < \infty, 0 < \theta < \pi, 0 < \phi < 2\pi \]

- Estimation of uncertainties via semi-classical approximations and the Uncertainty Principle
- Eigenfunction/eigenvalue relationships for different wave functions, including Parity operator \( \Pi \) defined to keep track of even and odd wavefunctions in potentials that are even:

\[ \Pi \psi(x) = \psi(-x) = \pm 1 \psi(x), \text{ with eigenvalues of } +1 (\text{even } \psi) \text{ and } -1 (\text{odd } \psi) \]

(E.g., see table on 1D problems in lecture slide)

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### Quantum Mechanics in 1 Dimension

- Types of solutions and boundary conditions arising for different potentials:
  - Particle in a rigid box
  - Particle in a soft box
  - Harmonic oscillator and application to molecular vibrations ...including Correspondence Principle limits
- Extension of rigid box to 3D by separation of variables:
  Degeneracy = several independent \( \psi \)'s yield same energy
- Be able to sketch qualitative form of a wave function in an arbitrary potential, assuming that it is piecewise constant, and using wavevector (k) or decay constant (\( \kappa \)) and Correspondence Principle arguments (e.g., \( |\psi|^2 \) is maximum where classical particle moves most slowly)
- Tunneling effects: general form of wavefunctions and boundary conditions in different regions and calculation of tunneling probability T(E) for special cases: high-wide barrier, alpha decay, tunnel diode, field emission, and scanning tunneling microscope (see lecture slides)
Hydrogenic Atom Wave Functions and Properties

- Separable form in r, θ, and ϕ: \( \psi_{n\ell m}(r, \theta, \phi) = R_n(r) \Theta_{\ell m}(\theta) \frac{1}{\sqrt{2\pi}} e^{im\phi} \)

- Quantum nos. n, \( \ell \), and m and their significance to energy, angular momentum, and parity eigenvalues:
  
  **Energy:**  \( \hat{H}\psi_{n\ell m} = E_n \psi_{n\ell m} \), \( E_n \) from Bohr formula  
  
  **Square of orbital angular momentum:**  \( \hat{L}^2 \psi_{n\ell m} = \hbar^2 \ell (\ell + 1) \psi_{n\ell m} \)

  **Z component of orbital angular momentum:**  \( \hat{L}_z \psi_{n\ell m} = \hbar m \psi_{n\ell m} \)

  **Parity:**  \( \hat{P} \psi_{n\ell m}(r, \theta, \phi) = \psi_{n\ell m}(r, \pi - \theta, \phi + \pi) = (-1)^\ell \psi_{n\ell m} \)

- Normalization and calculation of probabilities and expectation values, including integration with proper volume element in spherical polar coordinates:
  
  \( dV = r^2 \sin \theta \, dr \, d\theta \, d\phi \)

- Use of radial probability distribution  \( P_n(r) = \frac{2}{\pi a_n^2} r^2 R_n^2(r) \) to illustrate shell structure associated with a given n value, as well as different degrees of nuclear charge screening

- Graphical representation of different aspects of wave functions in 1D, 2D, and 3D, including radial and angular nodes

- Degeneracy of atomic \( \psi \)'s

- Vector model of quantization of space for orbital angular momentum

- Magnetic moment associated with orbital angular momentum

- Normal Zeeman effect: different degenerate \( m_\ell \) levels split by magnetic field

- Selection rules for dipole-radiation transitions and application to atomic spectra, x-ray spectra, and molecular rotational and vibrational excitations

Spin Angular Momentum, Spin Magnetic Moment

- Stern-Gerlach experiment and existence of spin \( \hat{s} \): one last relativistic effect!

- Adds quantum no. \( m_s = \pm 1/2 \) and spin magnetic moment  \( \mu_s = -\frac{e}{m} \hat{s} \)

- Additional eigenvalue properties with spin added are:

  **Square of spin angular momentum:**  \( \hat{S}^2 \psi_{n\ell m m_s} = \hbar^2 \left( \frac{1}{2} + 1 \right) \psi_{n\ell m m_s} = \frac{3}{4} \hbar^2 \psi_{n\ell m m_s} \)

  **Z component of spin angular momentum:**  \( \hat{S}_z \psi_{n\ell m m_s} = \hbar m_s \psi_{n\ell m m_s} = \pm \frac{\hbar}{2} \psi_{n\ell m m_s} \)

- Vector model of quantization of space for spin angular momentum

- Spin-orbit coupling and vector model of quantization of space for total angular momentum  \( \hat{j} = \hat{r} + \hat{s} \), with new quantum nos. of \( n, \ell, m, m_s \)

  **Square of total angular momentum:**  \( \hat{J}^2 \psi_{n\ell jm} = \hbar^2 j(j + 1) \psi_{n\ell jm} \), with \( j = 1/2, 3/2, 5/2, ... \)

  **Z component of spin angular momentum:**  \( \hat{J}_z \psi_{n\ell jm} = \hbar m \psi_{n\ell jm} \), with \( m = -j, -j+1, ..., j-1, j \)

- Spin-orbit interaction as an internal magnetic field created by nuclear motion around electron→current loop, and energy splittings (e.g. sodium emission/absorption doublet)
Wave Functions and Energies for Multielectron Atoms

-The essential points:
- the electron has intrinsic spin: \( s = \frac{1}{2} \), and z component linked to \( m_s = \pm \frac{1}{2} \)
- this leads to four total quantum nos. for non-relativistic electrons in atoms:
  \( n, \ell, m_{\ell}, m_s \)
- the wave function for a set of identical particles cannot yield results which depend on our choice of particle labelling: e.g., probability density:
  \[ |\psi(\ldots \tilde{r}_1 \ldots \tilde{r}_2 \ldots)|^2 = |\hat{P}_{12}\psi(\ldots \tilde{r}_1 \ldots \tilde{r}_2 \ldots)|^2 = |\psi(\ldots \tilde{r}_2 \ldots \tilde{r}_1 \ldots)|^2, \]
  where \( \hat{P}_{12} \) is the permutation operator which just interchanges the labels of all space and spin coordinates, thus "trading the places" of electrons 1 and 2 in \( \psi \).
- all particles are "fermions" or "bosons", depending on sign change when any two labels are interchanged. I.e., all valid many-particle wave functions are eigenfunctions of \( \hat{P}_{12} \), and all particles are in two groups:
  Fermions (electrons and all particles with half-integral spin = \( \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \ldots \)): antisymmetric \( \psi \)'s with -1 eigenvalue: \( \hat{P}_{12}\psi = -\psi \).
  Bosons (photons, all particles with integral spin = 0, 1, 2, 3,\ldots): symmetric \( \psi \)'s with +1 eigenvalue: \( \hat{P}_{12}\psi = +\psi \).

- This leads to the Pauli Exclusion Principle for electrons (or all Fermions):
  No two electrons in a multielectron atom can have all of \( n, \ell, m_{\ell}, m_s \) (or \( n, l, j, m_j \)) the same. (If they do, then \( \psi \) is trivially zero!)
- And also: Electrons with the same spin orientation (up, up) or (down, down) are never found at the same point in space, lowering their Coulomb repulsion ("exchange interaction"): explains magnetism and Hund’s First Rule.
- Inner-shell electron screening in multi-electron atoms lifts the H-atom degeneracy in \( \ell \) through an effective \( Z \) that varies with \( \ell \) for a given \( n \): energies always go as \( s < p < d < f \), with overlap of \( n \) values which e.g. makes 4s fill before 3d.
- Pauli Principle then leads to filling of atomic levels in Periodic Table. If open shell at the end, Hund’s First Rule says fill with maximum total spin \( S \). (Don’t worry about other Rules.)

Molecules

- “Electrons in a box” with one potential well on each atom; for diatomics—wave functions approximated as sums (bonding) and differences (anti-bonding) of atomic functions on each atom
- For polyatomic molecules, electronic wave functions describable as linear combinations of atomic orbitals on different atoms
- Diatomics also behave like 1D harmonic oscillators in their vibrations
- Diatomics also behave like rigid rotors in their rotations
- Examples of CO microwave absorption and ultraviolet photoelectron excitation, the microwave oven, and possible effects of cellphones on brain metabolism rate (via \( \Delta T \)?)

Maxwell Boltzmann, Fermi-Dirac and Bose-Einstein Statistics (Review)

- MB, FD, and BE statistics, basic formulas, similarities and differences
- Blackbody radiation as BE example, including solar spectrum, Cosmic microwave background.
- Electrons as FD example (see below in solids)
Lasers and Holography

- General principles of stimulated emission and population inversion (qualitative)
- Coherence (high monochromaticity) of laser light and application to holography (qualitative)
- Example of photoelectron holography (covered lightly, LBNL tour)

Solids and Solid State Devices

- Electron counting and level filling in solids
- Free-electron-like solids: density of states, Fermi energy, Fermi wave vector and velocity
- Li and Al as examples of nearly free-electron solids
- Periodic solids and Bloch functions as “universal” wave function form for solids
- The Kronig-Penney model and energy bands
- Semiconductor energy bands—e.g. Ge
- Metallic energy bands—e.g. non-magnetic Cu and ferromagnetic Fe
- Phenomenology of ferromagnetism and superconductivity/Cooper pairs
- Ionic solids and the Madelung sum
- Ionic energy bands—e.g. NaCl
- Semiconductor doping of n or p type
- The p-n junction diode, extension to the LED and the photovoltaic cell
- The metal-oxide-semiconductor field-effect transistor (MOSFET)
- Transistors and the logic gates of IT devices
- Nanoscience and nanotechnology
- Moore’s Law for integrated circuits; analogue for magnetic bit density in storage media

Nuclear Properties and Applications

- Decay of nuclei and particles and half lives
- Particles and anti-particles, the positron e.g.
- Notation including Z, N, and A and chemical symbol: $^AXN$
- Nuclear sizes, charge density versus radius, Rutherford scattering (review from 9HC and Chapter 4—two basic equations)
- Nuclear binding energies per nucleon, calculation of and stability versus mass number $A$
- The nuclear interactions: proton-proton Coulomb barrier plus strong nucleon-nucleon interaction plus hard-core repulsion
- Shell model (may give net spin on nucleus) and liquid drop model for calculating binding energies per nucleon
- Basic decay modes and meaning for changes in $^AXN$
- Applications to carbon dating, nuclear energy production by fission and fusion, understanding radiation from radon in earth’s crust
- Enrichment of U by diffusion and centrifugation, breeder reactors using Pu
- Nuclear spin, magnetic resonance (qualitative), Magnetic Resonance Imaging (MRI) in comparison to Positron Emission Tomography (PET)

Elementary Particles and Interactions

- Particles, antiparticles
- Leptons and Quarks, the table of elementary particles in the Standard Model and the manner in which quarks go together to make up common Hadrons = mesons (2 quarks, e.g. pion) + baryons (3 quarks, e.g., protons and neutrons)
- Four fundamental forces: electromagnetic, weak, strong, and gravitational, and four Bosons mediating them (photon; W, Z bosons; gluons; and gravitons, respectively)
- Dark matter (e.g. from gravitational lensing-covered earlier) and dark energy (e.g. from increase in rate of universe expansion)