GENERAL RELATIVITY: Einstein, 1915

Adds non-inertial (accelerated) ref. systems and gravity

Postulate = Principle of Equivalence: Gravitational mass is equal to inertial mass--or--observer cannot tell whether frame is accelerating or under influence of gravity
Light should be deflected (= accelerated) by gravity.

Principle of Equivalence
FIGURE 3.3. (a) Two blocks of wood, one with a spring attached and both having mass $m$, move with equal speeds $v$ and kinetic energies $K$ towards a head-on collision. (b) The two blocks collide, compressing the spring, which locks in place. The system now has increased mass, $M = 2m + 2K/c^2$, with the kinetic energy being converted into the potential energy of the spring.
1. Deflection of light by the sun’s gravitational field

→ "Gravitational lensing" by stars/galaxies

General relativity:
Experimental verifications:

FIRST OBSERVED IN 1919

\[ \theta \approx 0.0005^\circ = 0.00025^\circ \]

(Photon + Gravity) + 0.00025° (Curved Space-Time)
2. "Gravitational lensing" by stars/galaxies

Four images of star: “The Einstein Cross”

More discussion and great images at:
http://csep10.phys.utk.edu/astr162/lect/galaxies/lensing.html
“The Einstein Cross”—Possible Explanation

Quasar

Spiral Galaxy

Earth
"Gravitational lensing" by stars/galaxies--theory

T. Tyson, UCD--More details at:  http://www.lsst.org/Science/darkmatter2.shtml
"Gravitational lensing" by stars/galaxies

Also see effects of "dark matter" + "dark energy" \(\rightarrow\) accelerating expansion of universe: altogether 90-95% of the mass/energy in the universe!

More discussion and great images at:
http://csep10.phys.utk.edu/astr162/lect/galaxies/lensing.html
Visible matter

Figure 25.26 Dark Matter Map By measuring the distortions produced in the images of background objects and modeling the distribution of foreground dark matter required to account for these distortions, astronomers can produce maps of dark matter in the universe, which would otherwise be impossible to detect. (a) An optical view of a region of the sky containing a small galaxy cluster (too diffuse to be easily seen in this image, but easily detectable by its X-ray-emitting intracluster gas). The cluster is the clump of yellowish galaxies near the center of the frame. The fuzzy blue specks scattered across the frame are the much more distant background galaxies whose distortions are used to estimate the cluster's dark-matter content. (b) The distribution of dark matter in and near the visible cluster, obtained by analyzing the images of the background galaxies. Parts (a) and (b) are drawn to the same scale. (Deep Lens Survey team; data courtesy J. A. Tyson, Bell Labs, Lucent Technologies; NSF/NOAO)

J.A. Tyson, UCD Physics
Smaller Version of the Solar System Is Discovered

By DENNIS OVERBYE

Astronomers said Wednesday that they had found a miniature version of our own solar system 5,000 light-years across the galaxy — the first planetary system that really looks like our own, with outer giant planets and room for smaller inner planets.

"It looks like a scale model of our solar system," said Scott Gaudi, an assistant professor of astronomy at Ohio State University. Dr. Gaudi led an international team of 69 professional and amateur astronomers who announced the discovery in a news conference with reporters.

Their results are being published Friday in the journal Science. The discovery, they said, means that our solar system may be more typical of planetary systems across the universe than had been thought.

In the newly discovered system, a planet about two-thirds of the mass of Jupiter and another about 90 percent of the mass of Saturn are orbiting a reddish star at about half the distances that Jupiter and Saturn circle our own Sun. The star is about half the mass of the Sun.

Neither of the two giant planets is a likely abode for life as we know it. But, Dr. Gaudi said, warm rocky planets — suitable for life — could exist undetected in the inner parts of the system.

"This could be a true solar system analogue," he said.

Sara Seager, a theorist at the Massachusetts Institute of Technology who was not part of the team, said that "right now in exobiology, there is an inexorable path to finding other Earths." Dr. Seager praised the discovery as "a big step in finding out if our planetary system is alone.

Since 1995, around 250 planets outside the solar system, or exoplanets, have been discovered. But few of them are in systems that even faintly resemble our own. In many cases, giant Jupiter-like planets are whizzing around in orbits smaller than that of Mercury. But are these typical of the universe?

Almost all of those planets were discovered by the so-called wobble method, in which astronomers measure the gravitational tug of planets on their parent stars as they whirl around it. This technique can detect massive planets close to their stars.

The new discovery was made by a different technique that favors planets more distant from their star. It is based on a trick of Einsteinian gravity called microlensing. If, in the ceaseless shifting of the stars, two of them should become almost perfectly aligned with Earth, the gravity of the nearer one can stretch and magnify the light from the more distant one, causing it to get much brighter for a few days.

If the alignment is perfect, any big planets attending the nearer star will get into the act, adding their own little boosts to the more distant starlight.

That is exactly what started happening on March 28, 2006, when a star 5,000 light-years away in the constellation Scorpius began to pass in front of one 21,000 light-years more distant, causing it to flash. That was picked up by the Optical Gravitational Lensing Experiment, or OGLE.

Somewhat to the experimenters' surprise, by clever manipulation they were able to dig out of the data not just the masses of the interloper star and its two planets, but also rough approximations of their orbits, confirming the similarity to our own system. David P. Bennett, an assistant professor of astrophysics at the University of Notre Dame, said, "This event has taught us that we were able to learn more about these planets than we thought possible.

As a result, microlensing is poised to become a major new tool in the planet hunter's arsenal, "a new flavor of the month," Dr. Seager said.

Only six planets, including the new ones, have been discovered by microlensing so far, and the Scorpius event being reported Friday is the first in which the alignment of the stars was close enough for astronomers to detect more than one planet at once. Their success at doing just that on their first try bodes well for the future, astronomers say.

Alan Boss, a theorist at the Carnegie Institution of Washington, said, "The fact that these are hard to detect by microlensing means there must be a good number of them — solar system analogues are not rare."
2. Wavelength shifts in gravitational fields → time contraction and dilation

Also implies that observer on ground sees time passing faster above—reverse of time dilation in Special Relativity!
General relativity--Experimental verifications:

3. Escape from stars $\rightarrow$ Black Holes

If $\nu' \rightarrow 0$, \[ h\nu \approx \frac{GMh\nu}{c^2} R \rightarrow \]

Black Hole: \[ R_{\text{Schwartzchild}} = \frac{GM}{2c^2} \]

Inside!

\[ V(r) = -\frac{GMm_{\text{photon}}}{r} \]

\[ V(r) = -\frac{GM(h\nu / c^2)}{r} \]

\[ \therefore h\nu' = h\nu \left[ 1 \pm \frac{GM}{c^2} \left\{ \frac{1}{(r_1 \rightarrow R)} - \frac{1}{(r_2 \rightarrow \infty)} \right\} \right] = h\nu \left[ 1 \pm \frac{GM}{c^2 R} \right] \]
Had J. Robert Oppenheimer not led the US effort to build the atomic bomb, he might still have been remembered for conceiving of black holes. His 1939 Physical Review paper, written with graduate student Hartland Snyder, described how a star might collapse into an object so dense that not even light could escape its gravitational clutches. The paper was hardly noticed until the 1960s, when astrophysicists began to seriously consider that such extreme objects might exist. John Wheeler of Princeton University then came up with the name "black holes" for these now standard elements of astrophysics.

(J. R. Oppenheimer and H. Snyder, Phys. Rev. 56, 455 (1939))

Link to the paper: http://link.aps.org/abstract/PR/v56/p455
COMPLETE Focus story at http://focus.aps.org/story/v13/st23
The first good candidate for a black hole—Cygnus X-1

**FIGURE 21.9** The brightest star in this photograph is a member of a binary system whose unseen companion, called Cygnus X-1, is thought to be a good candidate for a black hole. (The rectangle outlines the field of view illustrated in the next figure.)

(7,000 LIGHT-YRS.) AWAY

**FIGURE 21.10** Invisible x-rays emitted near the Cygnus X-1 source can be analyzed by changing the detected x-rays into electronic signals, which can then be viewed on a video screen from which this picture is taken. (The field of view here is outlined by the rectangle in the previous figure.)

FROM "THE UNIVERSE" BY CHAISON
FIGURE 21.11 Artist's conception of a binary system containing a large, bright, visible star, and an invisible, x-ray emitting black hole. This particular drawing depicts the Cygnus X-1 region discussed in the text.
...And the center of the Milky Way Galaxy (and probably all other galaxies?) has a black hole:

\[ M_{\text{Milky Way}} \approx 3 \times 10^6 \, M_{\odot} \]
2. Wavelength shifts in gravitational fields—more accurate view

\[ V(r) = -\frac{GMm_{\text{photon}}}{r} = -\frac{GM(h\nu / c^2)}{r} \]

\[ \therefore \quad h\nu' = h\nu \left[ 1 \pm \frac{GM}{c^2} \left\{ \frac{1}{(r_1 \to R)} - \frac{1}{(r_2 \to \infty)} \right\} \right] = h\nu \left[ 1 \pm \frac{GM}{c^2 R} \right] \]
(See disc. of Fig. 2.20-Special & Example 15.1-Due to General, clocks in airplanes are both too fast as measured on the ground at the end of the flight)

**Special:**
- Time dil.
- Faster: 96 ns, 179 ns*
- Slower: 184 ns, 140 ns

**General:**
- Gravity @ $10^4$ m
- 48.6 flight hours: 275±21 ns, 273±7 ns
- 41.2 flight hours: 40±23 ns - Theory, 59±10 ns - Expt.

* Slightly different paths around the earth in latitude/longitude
Where are the 24 GPS satellites?

The 24 GPS satellites (21 active, 3 spare) are in orbit at 10,600 miles above the earth. The satellites are spaced so that from any point on earth, four satellites will be above the horizon. Each satellite contains a computer, an atomic clock, and a radio. With an understanding of its own orbit and the clock, the satellite continually broadcasts its changing position and time. On the ground, any GPS receiver contains a computer that "triangulates" its own position by getting bearings from three of the four satellites. The result is provided in the form of a geographic position—longitude and latitude—to, for most receivers, within a few meters.
Time accuracy among all satellite+earth clocks must be $\leq 20 \times 10^{-9}$ s

→ time for light to travel $(3.0 \times 10^8)(20 \times 10^{-9}) = 6$ m

→ overall GPS accuracy of 5-10 m

Special Relativity:

$v = 14,000$ km/hr; seconds per day = $86,400$ s; $\beta = (1.4 \times 10^7$ m-hour$^{-1})/(3.6 \times 10^3$ s-hour$^{-1}$) $(3.0 \times 10^8$ ms$^{-1}) = 0.130 \times 10^{-4}$; $\gamma \approx 1 + 0.5 \beta^2 = 1.0000000000845$; therefore per day, clock in satellite is slower by $(8.45 \times 10^{-11})(8.64 \times 10^4) = 7.3 \times 10^{-6}$ s per day as measured on the ground

General Relativity: assume $g$ constant

$H = 20,000$ km;
$h \nu_{sat}(1+gH/c^2) = h \nu_{ground}$
$\Delta t_{sat}/\Delta t_{ground} = (1+gH/c^2) = 1 + (9.8$ ms$^{-2})(2.0 \times 10^7$ ms$^{-1})/(3.0 \times 10^8)^2$
$1+2.17 \times 10^{-9}$;
therefore per day, clock in satellite is faster by $(2.17 \times 10^{-9})(8.64 \times 10^4) = 18.7 \times 10^{-5}$ s
$= 187 \times 10^{-6}$ s per day* as measured on the ground
Relativity and GPS systems (cont’d.)

General Relativity: more accurate

\( H = 20,000 \text{ km} \gg R = 6,380 \text{ km} \)

Seconds per day = 86,400

\[
\frac{h_{\text{sat}}}{h_{\text{ground}}} = \Delta t_{\text{sat}} / \Delta t_{\text{ground}} = 1 + \frac{GM}{c^2} \left[ \frac{1}{r_1} - \frac{1}{r_2} \right] = 1 + \frac{(6.67 \times 10^{-11})(6.0 \times 10^{24} \text{ kg})/(3.0 \times 10^8)^2 x}{1/6,380,000 - 1/26,380,000} = 1 + 4.44 \times 10^{-3} \left[ 1.56 \times 10^{-7} - 0.38 \times 10^{-7} \right] = 1 + 5.2 \times 10^{-10} ;
\]

therefore per day, clock in satellite is faster by

\[(5.20 \times 10^{-10})(8.64 \times 10^4) = 45.3 \times 10^{-6} \text{ s per day}, \text{ as measured on the ground} \]

So must correct for both Special and General effects for GPS to have a chance of working!

*See also discussion at http://www.astronomy.ohio-state.edu/~pogge/Ast162/Unit5/gps.html

More accurate calc. gives 45 \times 10^{-6} \text{ s for General}
General relativity:  
Experimental verifications:  

4. Precession of the perihelion of Mercury:  
Distortion of spacetime due to Sun’s gravity

Newtonian mech. predicts ~ 5557 arcseconds/century = 5557/3600 = 1.6°/century, ~43 arcsec/century = 0.011°/century smaller than observed.

Figure 3.7 Precession of the perihelion (point of closest approach to the sun) of the orbit of Mercury. Here the eccentricity of the elliptic orbit is greatly exaggerated, as is the rate of precession.
5. Gravity waves?

General relativity - Experimental verifications:

- Evidence for Gravity Waves?
  (Nobel Prize, 1994)

**Binary Pulsar = Two Neutron Stars**

- PSR 1913 + 16
- Orbital Period ≈ 7.8 Hours
  Decreasing

**Orbit Phase Shift (s)**

- PSR 1913 + 16

**Figure 3.9** The orbital phase shift of the binary pulsar PSR 1913 + 16 as a function of time. The points represent experimental data obtained over one decade. The solid line is the calculated phase shift based on the theory of general relativity. *(After Modern Physics in America, American Institute of Physics, 1988, p. 80.)*

*(See Physics Today, April, 1993, p. 9)*
Once upon a time life was simple, science was small, and one could aspire to keep up with physics. Today keeping up requires more energy and time than most of us can muster. That, at any rate, is my excuse for not paying attention to general relativity as it blossomed from a curiosity into a hard science.

For 40 years general relativity was in the peculiar position of being the best known but the least verified theory in physics. The myth of general relativity was born in 1919 when Arthur Eddington announced to the world that he had observed the deflection of starlight by the Sun's gravitational field. Einstein became an instant celebrity, and general relativity was elevated to its mystical pedestal. The deflection of light, however, was barely discernible. There was one other strand of evidence—the precession of the perihelion of Mercury. But the precession rate was tiny and had to be extricated from effects of planetary perturbations. Theory does not flourish without experiment, and even as Broadway lyricists waxed poetic about general relativity (“Your charm is not that of Circe’s with her swine / Your brain would never deflate the great Einstein”—Cole Porter), as did poets (“lenses extend unwish through curving wherewhen till unwish returns on its unself”—e. e. cummings), scientifically it remained little more than a curiosity. The Earth’s field. Joseph Weber started the quest to detect gravitational radiation. Irwin Shapiro, having realized that according to general relativity, gravity retards light, applied the newly created field of planetary radar to study it and carried out a generation of experiments on general relativistic effects using satellites, planets and stellar radio sources. Theory and experiment were in harmony, but the range of phenomena was narrow and the effects were small.

In 1974 Nature suddenly started to flaunt general relativity. Joseph Taylor and Russell Hulse, working at Delay in the time of periastron of the binary pulsar PSR 1913 + 16 as tracked over the years. The blue curve is the time delay due to the loss of energy by gravitational radiation; the curve is calculated from general relativity using the binary pulsar's measured properties, with no adjustable parameters. (Adapted from J. H. Taylor, in Proc. IVth Rencontres de Blois, T. Tranh, ed., IOP Publishing, Bristol, England, in press.)
General relativity-
-Experimental verifications:

The LIGO experiment, just getting underway: Interferometric measurements of gravity waves = gravitational distortion of space time:

http://www.ligo.caltech.edu/LIGO_web/about/brochure.html
General relativity - Gravity waves from the biggest shows possible: two black holes colliding

http://www.aip.org/png/2006/256.htm

Number 771 #1, March 29, 2006 by Phil Schewe and Ben Stein

Black Hole Merger Movie

Accurate calculations of the gravitational waveforms emitted during the collision of black holes can now be made. A new computer study of how a pair of black holes, circling each other, disturbs the surrounding space and sends huge gusts of gravitational waves outwards, should greatly benefit the experimental search for those waves with detectors such as the Laser Interferometer Gravitational-Wave Observatory (LIGO) and the planned Laser Interferometer Space Antenna (LISA).

The relative difficulty of computer modeling of complicated physical behavior depends partly on the system in question and on the equations that describe the forces at work. To describe the complicated configuration of charges and currents, one uses Maxwell's equations to determine the forces at work. In the case of black-hole binaries, the equations are those from Albert Einstein's theory of general relativity. Black holes encapsulate the ultimate in gravitational forces, and this presents difficulties for computations attempting to model behavior nearby. Nevertheless, some physicists at the University of Texas at Brownsville have now derived an algorithm that not only produces accurate estimates of the gravity waves of the inspiraling black holes, even over the short time intervals leading up to the final merger, but also is easily implemented on computers (see figures and movie at Physics News Graphics).

"The importance of this work," says Carlos Lousto, one of the authors of the new study, "is that it gives an accurate prediction to the gravitational wave observatories, such as LIGO, of what they are going to observe."

The new results are part of a larger study of numerical relativity carried out at the University of Texas, work referred to as the Lazarus Project. Figures and movie at Physics News Graphics.

Contact: Carlos Oscar Lousto, lousto@phys.utb.edu, 956-882-6651

Back to Physics News Update
6. Frame dragging

Stanford ready to give Einstein big exam
Satellite experiment will test whether General Theory of Relativity is correct

See Section 15.5 of textbook, and http://sfgate.com/cgi-bin/article.cgi?f=/c/a/2004/04/05/MNG0K69C151.DTL&type=news

http://einstein.stanford.edu/
GENERAL RELATIVITY: Einstein, 1915

Adds non-inertial (accelerated) ref. systems and gravity

Postulate = Principle of Equivalence: Gravitational mass is equal to inertial mass--or--observer cannot tell whether frame is accelerating or under influence of gravity

Consequences:
- Light (photons) attracted by gravity → black holes and gravitational lensing → "seeing" dark matter
- Time contraction or dilation between clocks at different points in gravitational potential (planes, GPS)
- Motion of Mercury orbit (perihelion)
- Gravity waves? Probably observed
- Frame dragging? Now observed