Physics 9HE-Modern Physics
Midterm
February 5, 2009
(100 points total)
You may tear off this sheet.

Miscellaneous data and equations:

c = 3.00 x 10^8 m/s  e = 1.60 x 10^{-19} C  1 eV = 1.60 x 10^{-19} J  1 A = 10^{-10} m
M_{Sun} = 2 x 10^{30} kg  M_{Earth} = 5.98 x 10^{24} kg  r_{Earth} = 6.38 x 10^6 m
m_e = 9.1094 x 10^{-31} kg = 0.5110 MeV/c^2  m_p = 1.6726 x 10^{-27} kg = 938.27 MeV/c^2
m_n = 1.6749 x 10^{-27} kg = 939.57 MeV/c^2  m(1H) = 1.0078 u
G = 6.67 x 10^{-11} Nm^2/kg^2  g = 9.81 m/s^2  \sigma = 5.67 x 10^{-8} Wm^{-2}K^{-4}
\hbar = 6.63 x 10^{-34} Js  \eta = h/2 \pi  \pi = 1.05 x 10^{-34} J-s
h = 6.63 x 10^{-34} Js  k_B = 1.38 x 10^{-23} JK^{-1}  a_0 = 0.529 \AA

N(t) = N_0 e^{-t/\tau_0} = N_0 e^{-0.693t/\tau_{1/2}}  k_C \equiv 1/(4\pi\varepsilon_0) = 8.98 x 10^9 Nm^{-2}C^{-2}  RH = 1.09678 x 10^7 m^{-1}

\rho = \gamma m\bar{u}  E = \gamma m c^2 = K + m c^2  E^2 = p^2 c^2 + m^2 c^4
E = pc = h\nu  \Delta \nu = \Delta T / T \approx -GM / c^2 \left[ 1 - \frac{1}{r_1} - \frac{1}{r_2} \right] \approx -\frac{gH}{c^2} \Delta \nu \approx -\frac{\Delta \lambda (if \Delta \nu \ll 1)}{\lambda (if \Delta \nu \ll 1)}  R_{Sch} = \frac{2GM}{c^2}

\lambda_{max}T = 2.898 x 10^{-3} mK  I(\lambda, T) = \frac{2\pi c^2 h}{\lambda^5} \cdot \frac{1}{e^{h\nu/kT} - 1}  R(T) = \varepsilon \sigma T^4

h\nu = K_{max} + \phi  n\lambda = 2\sin \theta  F_{coul} = k_C \frac{q_1 q_2}{r^2}  F_{radial} = \frac{mv^2}{r}
\rho = \frac{4\pi\varepsilon_0 \hbar^2}{m_ne^2} = 0.529 \AA

\lambda = h/p  p = \hbar k  \Delta x \Delta p_x \geq \hbar/2  \Delta E \Delta t \geq \hbar/2  \nu_{ph} = \bar{\omega}/\hbar  \nu_{gr} = d\omega/dk

e^{ix} = \cos x + i \sin x  \cos x = \frac{1}{2} \left[ e^{ix} + e^{-ix} \right]  \sin x = \frac{1}{2i} \left[ e^{ix} - e^{-ix} \right]

\sin 2t = 2 \sin t \cos t  \cos 2t = \cos^2 t - \sin^2 t  \cos^2 t - 1 = 1 - 2 \sin^2 t

\psi(x) = \frac{a_0}{2} + \sum_{n=0}^{\infty} a_n \cos(\frac{2\pi n}{\ell} x) + \sum_{n=0}^{\infty} b_n \sin(\frac{2\pi n}{\ell} x), \quad \text{with } k_n = \frac{2\pi n}{\ell}, \quad \text{and}

a_n = \text{aver. of } \psi \text{ over } \ell, \quad a_n = \frac{2}{\ell} \int_0^\ell \psi(x) \cos(\frac{2\pi n}{\ell} x)dx, \quad b_n = \frac{2}{\ell} \int_0^\ell \psi(x) \sin(\frac{2\pi n}{\ell} x)dx

\psi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} c(k) e^{ikx} dk, \quad \text{with } c(k) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \psi(x) e^{-ikx} dx

\hat{H} \Psi = i\hbar \frac{\partial \Psi}{\partial t} \quad \hat{H} = \hat{K} + V  \Psi = \psi e^{-\frac{iEt}{\hbar}}  \psi = e^{-\frac{iEt}{\hbar}}

\hat{\rho}_x = -i\hbar \frac{\partial}{\partial x}  \hat{K}_x = -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2}  \Delta A = \sqrt{\langle A^2 \rangle - \langle A \rangle^2} \rightarrow \hat{A}_\psi = a_\psi \rightarrow A = \int \psi* \hat{A}\psi dx
\[ k = \sqrt{\frac{2m(E-V)}{\hbar^2}} \quad \psi \propto e^{\pm ikx} \quad \psi \propto \sin(kx), \cos(kx) \quad \kappa = \alpha = \frac{1}{\delta} = \sqrt{\frac{2m(V-E)}{\hbar^2}} \quad \psi \propto e^{\pm \kappa x} \]

\[ \psi_n = \left( \frac{2}{L} \right)^{1/2} \sin \left( \frac{n\pi x}{L} \right) \quad E_n = \frac{\pi^2 \hbar^2 n^2}{2mL^2} \]

\[ \psi_n = H_n(x)e^{-\alpha x^2/2} \quad \alpha = \frac{m\kappa}{\hbar^2} \quad \omega = \sqrt{\frac{\kappa}{m}} \quad E_n = \left( n + \frac{1}{2} \right) \hbar \omega \]

---Tear off this sheet and begin exam---
Physics 9HE-Modern Physics
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(100 points total)

Name (printed)_______________________________________

Name (signature)_______________________________________

Student ID No._______________________________________

Are you interested in going on a Saturday tour of the Lawrence Berkeley National Laboratory? Yes_______No_______.
If yes, which date(s) are OK?: Feb 14______Mar 7______

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[1] (30 Points)
In a classic experiment often referred to as the Harvard Tower Experiment, gamma rays of 14.4 keV energy from an Fe$^{57}$ source mounted on a sinusoidally vibrating speaker were directed upward toward an Fe$^{57}$ absorbing detector 22.6 m above it, as shown in the diagram below. The transition involved in the gamma ray also yields an extremely narrow energy spread of about $10^{-8}$ eV, so the gamma ray energy has to be matched to this degree at the detector to be absorbed.
(a) At what velocity in the up/down motion of the speaker can gamma ray absorption occur?

The general relativity effect of gravity on light will lower the frequency of the upward moving photon

\[ \frac{\Delta v_{GR}}{v_0} \approx \frac{gH}{c^2} \]

so the speaker has to be moving toward the detector to compensate this via the Doppler effect, which gives

\[ \nu = v_0 \left( \frac{1 \pm \beta}{\sqrt{1 \pm \beta}} \right) \equiv v_0 [1 \pm \beta] \text{ for } \beta \ll 1, \text{ or } \frac{\Delta v_{Dopp}}{v_0} = \left[ \frac{v + v/c}{v_0} \right]. \]

In order for the effects to cancel then,

\[ \nu \frac{\Delta v_{GR}}{v_0} + \frac{\Delta v_{Dopp}}{v_0} = 0, \text{ which implies } \frac{gH}{c} = \nu = (9.81 \text{ m/s}^2)(22.4 \text{ m}) / (3.00 \times 10^8 \text{ m/s}) \]

\[ = 73.2 \times 10^{-6} \text{ m/s} = 0.0732 \text{ mm/s}. \]

(b) Estimate the lifetime of the excited state in Fe^{57} that is involved in this experiment.

Just use the Uncertainty Principle in time and energy, with \( \Delta t \) identified as the state lifetime, as

\[ \Delta E \Delta t \approx \Delta E \tau = \hbar / 2 \approx \hbar, \]

so

\[ \tau = \text{lifetime} = \frac{1.05 \times 10^{-34} \text{ J-s}}{2(10^{-9} \text{ eV})(1.60 \times 10^{-19} \text{ J/eV})} = 0.01561 \times 10^{-7} = 1.56 \times 10^{-9} \text{ s}, \text{ or 2 times this} \]

without the numerator 2, or \( 3.1 \times 10^{-9} \text{s} \). So any no. in this range OK.

[2] (10 points)

Greater use of solar energy is one of the big promises for the future. To estimate the magnitude of this source, assume that the sun is a spherical blackbody of diameter \( 1.39 \times 10^9 \) m at 6000 K, and that its distance from the earth is \( 1.50 \times 10^{11} \) m. Estimate the total radiant power incident per square meter of surface on the earth that looks directly at the sun.
At the sun's surface, the radiated power, assuming an ideal blackbody with $\varepsilon = 1$, will be given by
\[ R_{\text{SunSurface}}(T) = \sigma T^4. \]
This is radiated from the surface in radial directions, and conservation of energy requires that it will decay per unit area according to $1/r^2$. Thus, at the earth's surface it will be
\[ R_{\text{EarthSurface}}(T) = \sigma T^4 \left[ \frac{r_{\text{sun}}}{r_{\text{earth}}} \right]^2 = (5.67 \times 10^{-8} \text{ W/m}^2\text{K}^4)(6000)^4 \left[ \frac{0.7 \times 10^9 \text{ m}}{1.50 \times 10^{11} \text{ m}} \right]^2 \]
\[ = (5.67 \times 10^{-8})(1.30 \times 10^{15})(0.218 \times 10^{-4}) \]
\[ = 1.60 \times 10^3 \text{ W/m}^2 = 1600 \text{ W/m}^2 \text{ at the equator.} \]
In No. California with a latitude of about $40^\circ$, this gets reduced by $\cos(\text{latitude}) = 0.76$ to yield $= 1200$, and reflection and absorption by the atmosphere knock it down to the about 1000 W/m$^2$ that one often sees quoted.

[3] (30 points)
Answer the following questions concerning the approximate cosine/sine series representation of the square wave shown below, whose 7th order coefficients have been calculated and are also summarized in the figure. Refer to the equations given at the beginning of the exam.

(a) Why are all the $a_k$ coefficients equal to zero?
The square wave is an odd function, and cosines are even. Thus they cannot contribute.
(b) Why are \( b_2, b_4, \) and \( b_6 \) equal to zero? Illustrate this with a sketch.

Even sine terms, don’t have the symmetry of the square wave in each distinct half cycle, but more quantitatively, they yield equal + and – terms in the integration within each half-cycle in calculating \( b_k \), as illustrated above.

(c) Calculate \( b_1 \).

\[
b_1 = \frac{2}{\ell} \int_0^{\ell} \psi(x') \sin \left( \frac{2\pi}{\ell} x' \right) dx' = 2 \frac{2}{\ell} \int_0^{\ell/2} 1 \times \sin \left( \frac{2\pi}{\ell} x' \right) dx'
\]

or, with change of variables to \( \frac{2\pi}{\ell} x' = x'' \), \( dx' = \frac{\ell}{2\pi} dx'' \), \( x' = \ell / 2 \rightarrow x'' = \pi \)

\[
2 \frac{2}{\ell} \int_0^{\ell/2} \sin(x'') dx'' = \frac{1}{\pi} \left[ \cos(x'') \right]_0^{\ell/2} = \frac{1}{\pi} [1 - (-1)] = \frac{4}{\pi} = 1.273
\]

(which agrees with the website value at: http://www.falstad.com/fourier/index.html).
[4] (30 points)

Consider an electron trapped inside the potential well shown below, and assume that it can have an energy of 200 eV, as indicated:

(a) Indicate as precisely as you can the functional forms of the wave function in each of regions 1-4, including the boundary conditions required at 0, 50 and 100 Angstroms.

(b) Now qualitatively sketch on the diagram the form of the wavefunction in each of the regions 1-4 indicated, being careful to show the relative wavelengths and the relative amplitudes in each region. That is, indicate in which region of the box the particle is most likely to be found?

(c) Indicate the classical turning points appropriate to this state on the diagram as well.

See drawing below for all of (a)-(c).

\[ \Psi_1 = 0, \text{ due to } \infty \text{ potential, } \Psi_2(x) = A \sin k_2 x, \text{ with no } \cos k_2 x \text{ term due to boundary condition.} \]

\[ \Psi_3(x) = B \sin k_3 x + C \cos k_3 x, \Psi_4(x) = D \exp(-k_4 x) \]

with boundary conditions of:

1 - 2: \[ \Psi_2(0) = 0, \text{ no conditions on derivative} \]

2 - 3: \[ A \sin k_2(50) = B \sin k_3(50) + C \cos k_3(50) \]

\[ k_2 A \cos k_2(50) = k_3 B \cos k_3(50) - k_3 C \sin k_3(50) \]

3 - 4: \[ B \sin k_3(100) + C \cos k_3(100) = D \exp(-k_4(100)) \]

\[ k_3 B \cos k_3(100) - k_3 C \sin k_3(100) = -k_4 D \exp(-k_4(100)) \]

and \[ k_2 = \sqrt{\frac{2m_e(200)}{\hbar^2}} = \sqrt{\frac{2(9.11 \times 10^{-31} \text{kg})(200 \text{eV})(1.60 \times 10^{-19} \text{J/eV})}{(1.05 \times 10^{-34})^2}} = 72.7 \times 10^6 \text{m}^{-1}, \]

\[ k_3 = \sqrt{\frac{2m_e(50)}{\hbar^2}}, \quad k_4 = \sqrt{\frac{2m_e(100)}{\hbar^2}} \]
(d) For the energy values shown on the diagram, what is the wavelength of the particle in region 2? You may do this problem non-relativistically. See no. above for $k_2$. From this, 
\[ \lambda_2 = \frac{2\pi}{k_2} = \frac{0.0864 \times 10^{-9}}{m} = 0.864 \text{ Å}. \]