5—Wave properties of matter

<table>
<thead>
<tr>
<th>All sections, plus supplementary reading on Fourier analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probs. in Chap. 5: 1, 13, 10, 11, 12, 17, 19, 22, 26, 27, 28, 33, 40, 43, 45, 48, 54, plus Special Problems: S1-With reference to the website: <a href="http://www.falstad.com/fourier/index.html">http://www.falstad.com/fourier/index.html</a>, calculate the first two non-zero Fourier coefficients in the cosine+sine series representing a square wave, and show that they agree with the numbers given at this site. Why are there no cosine terms? S2-With reference to the supplementary reading on Fourier integrals, show that the final formula for $g(\omega)$ on p. 3 and plotted in 4.20 is correct.</td>
</tr>
</tbody>
</table>

6—Quantum theory

<table>
<thead>
<tr>
<th>Sections 6.1-6.4, then all remaining sections</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probs. in Chap. 6: S1-Show that the wave function for a free particle traveling to the right is an eigenfunction of the momentum operator $\hat{p}$ and the kinetic energy operator $\hat{K}$, 2, 5, 7, 8, 9, 11, 15, 20, 23, 26, 28, 32, 37, 40, 41, 47, 54</td>
</tr>
</tbody>
</table>

Midterm through Chapter 6, Lecture Slides 4, other material covered in lecture
The Postulates of (Non-Relativistic) Quantum Mechanics
(The Rules of the Game)

- **Everything** we can know about the motion of a particle = a matter wave is contained in the wave function $\Psi$:

1D: $\Psi(x,t)$

3D: $\Psi(x,y,z,t)$ or $\Psi(r,\theta,\phi,t)$

and $\Psi$ may be a complex quantity:

$\Psi = \text{Re}\Psi + i\text{Im}\Psi$

- $\Psi$ is determined from the time-dependent Schroedinger Equation:

1D: $-\frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x^2} + V(x)\Psi = i\hbar\frac{\partial\Psi}{\partial t}$

3D: $-\frac{\hbar^2}{2m}\left[\frac{\partial^2\Psi}{\partial x^2} + \frac{\partial^2\Psi}{\partial y^2} + \frac{\partial^2\Psi}{\partial z^2}\right] + V(x,y,z)\Psi = i\hbar\frac{\partial\Psi}{\partial t}$

which can be simplified, if $V$ indep. of $t$, using separation of variables to the time-independent Schroedinger Equation:

1D: $\Psi(x,t) = \psi(x)e^{-iEt/\hbar} = \psi(x)e^{-i\omega t}$

$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi(x)}{\partial x^2} + V(x)\psi(x) = E\psi(x)$$

3D: $\Psi(x,y,z,t) = \psi(x,y,z)e^{-iEt/\hbar} = \psi(x,y,z)e^{-i\omega t}$

$$-\frac{\hbar^2}{2m}\left[\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi}{\partial z^2}\right] + V(x,y,z)\psi(x,y,z) = E\psi(x,y,z)$$
The Postulates (cont’d)

• To derive information from \( \psi \), we use the following recipes:

1D: Probability of finding particle in \( x \) to \( x+dx \) = \( |\psi|^2 dx = \psi^* \psi dx \)

3D: Probability of finding particle in \( x \) to \( x+dx \), \( y \) to \( y+dy \), \( z \) to \( z+dz \) = \( |\psi|^2 dx dy dz = \psi^* \psi dx dy dz \)

Therefore, \( \psi \) must be “normalizable” such that:

1D : \( \int_{-\infty}^{\infty} |\psi(x)|^2 \, dx = \int_{-\infty}^{\infty} \psi^*(x)\psi(x) \, dx = 1 = \text{total probability} \)

3D : \( \iiint_{\text{All space}} |\psi(x,y,z)|^2 \, dx \, dy \, dz = \iiint_{\text{All space}} \psi^*(x,y,z)\psi(x,y,z) \, dx \, dy \, dz = 1 = \text{total probability} \)

and \( \psi \) must be well-behaved such that:

\[
\psi \begin{cases} 
\text{must be finite everywhere, if a bound state also go to zero at } \pm \infty \\
\text{must be continuous; as must also be } d\psi/dx, (d\psi/dy, d\psi/dz) \text{--unless potential is infinite} \\
\text{must be a single-valued function of } x, (y, z) 
\end{cases}
\]
The Postulates (cont’d)

• To every observable quantity $A$ (like $p$, $K$, total $E$) there corresponds a Q.M. operator $\hat{A}$:

$$\hat{p}_x = -i\hbar \frac{\partial}{\partial x}, \quad \hat{K}_x = -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2}, \quad \hat{E} = i\hbar \frac{\partial}{\partial t}.$$

• Each time we measure $A$, we get some value “$a$” that is one of the “eigenvalues” of $\hat{A}$, as defined from

$$\hat{A}\phi_a = a\phi_a, \text{with } \phi_a = \text{an "eigenfunction" of } \hat{A}.$$  
That is, the measurement of $A$ forces $\psi$ to give us one of these values

• The average value over many measurements = “expectation value” of an observable $A$ is given by:

1D: $\langle A \rangle = \int_{-\infty}^{\infty} \psi^*(x) \hat{A} \psi(x) dx$

3D: $\langle A \rangle = \iiint_{\text{All space}} \psi^*(x,y,z) \hat{A} \psi(x,y,z) dxdydz$
The Postulates (cont’d)

Some important observables and their operators are:

<table>
<thead>
<tr>
<th>Observable</th>
<th>Operator</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>$\hat{A}$</td>
</tr>
</tbody>
</table>

Space coord.:  
- $x$ $\rightarrow$ $x$
- $y$ $\rightarrow$ $y$
- $z$ $\rightarrow$ $z$

Momentum:  
- $p_x$ $\rightarrow$ $\hat{p}_x = -i\hbar \frac{\partial}{\partial x}$
- $p_y$ $\rightarrow$ $\hat{p}_y = -i\hbar \frac{\partial}{\partial y}$
- $p_z$ $\rightarrow$ $\hat{p}_z = -i\hbar \frac{\partial}{\partial z}$

Kinetic energy:  
- $K_x$ $\rightarrow$ $\hat{K}_x = -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2}$
- $K_y$ $\rightarrow$ $\hat{K}_y = -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial y^2}$
- $K_z$ $\rightarrow$ $\hat{K}_z = -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial z^2}$

Total energy:  
- $E$ $\rightarrow$ $\hat{E} = i\hbar \frac{\partial}{\partial t}$

Potential energy:  
- $V(x,y,z)$ $\rightarrow$ $V(x,y,z)$

Hamiltonian:  
- $H$ $\rightarrow$ $\hat{H} = \hat{K} + V$

$\therefore$ Schroedinger Equations also:
- $\hat{H}\Psi = i\hbar \frac{\partial \Psi}{\partial t}$
- $\hat{H}\psi = E\psi$
Solving and Understanding a Q.M. Problem

(One-dimensional example)

• Set up the classical problem, including $V(x) = \text{potential}$

• Turn it into a Q.M. problem using the time-independent Schroedinger Equation:

$$\hat{H}_\psi = \hat{K}_\psi + V_\psi = E_\psi, \text{ or } -\frac{\hbar^2}{2m} \frac{d^2 \psi(x)}{dx^2} + V(x)\psi(x) = E\psi(x)$$

• Solve the resulting differential equation, if necessary using different solutions in regions of different potential. This yields all possible mathematical solutions.

• Choose physically reasonable solutions if possible. E.g., $\psi$ cannot be infinite anywhere and it will be zero if the potential is infinite (as in rigid box).

• Apply boundary conditions at edges of problem or between regions of different potential like 1 and 2 below:

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>$x=a$</td>
<td>$x=a$</td>
</tr>
<tr>
<td>$\psi_1(a)$</td>
<td>$\psi_2(a)$</td>
</tr>
<tr>
<td>$d\psi_1/dx(a)$</td>
<td>$d\psi_2/dx(a)$</td>
</tr>
</tbody>
</table>

This usually gives the allowed quantized energies $E = E_n$, even before getting the final answer for $\psi$. 
• Normalize ψ via: \[ \int_{-\infty}^{\infty} \psi^* \psi \, dx = 1 \]

• Calculate observable quantities like:
  
  Probability density: \[ \psi^* \psi \]

  Expectation values of different operators:
  \[ <A> = \int_{-\infty}^{\infty} \psi^* \hat{A} \psi \, dx = \text{average value over many measurements} \]

• Add time dependence if needed, via
  \[ \Psi(x,t) = \psi(x) e^{-iEt/\hbar} = \psi(x) e^{-i\omega t} \]

• Ask about:
  
  **Eigenfunction properties**--Does this hold for any operator? If \[ \hat{A} \psi = a \psi, \text{ where } a \text{ is some number} \]
  then \( \psi \) is an eigenfunction of this operator with "a" as eigenvalue, and we will always measure the same value for the observable associated with A. That observable is thus "sharp", with no inherent uncertainty: \( \Delta A = 0 \).

  **Consistency with uncertainty principles**-- How is this wave function consistent with uncertainty principles, esp. those for position and momentum?

  **Correspondence Principle Limit**--Think about what happens in the limit of large quantum nos. and make reference to the equivalent classical problem.
THE "FREE" PARTICLE - THE SIMPLEST PROBLEM:

**CLASSICALLY:**
\[
\frac{\vec{p}^2}{m} \rightarrow \text{cons.}, \quad E = k = \frac{p^2}{2m} = \text{cons.} \quad \Rightarrow \quad V(x) = \text{cons.} = 0
\]

**QUANTUM MECHANICALLY:** SOLVE t-INDEP. SCHROED. EQU.
\[
-\frac{h^2}{2m} \frac{d^2\psi(x)}{dx^2} + V(x)\psi(x) = E\psi(x) \Rightarrow \frac{d^2\psi(x)}{dx^2} = -\frac{2mE}{h^2}\psi(x)
\]

Try:
\[
\psi = Ae^{bx}, \quad \frac{d\psi}{dx} = bAe^{bx} = b\psi, \quad \frac{d^2\psi}{dx^2} = b^2Ae^{bx} = b^2\psi
\]

\[b^2\psi = -\frac{2mE}{h^2}\psi \Rightarrow E = -\frac{b^2h^2}{2m} = \text{no.}, \quad \text{so} \quad b^2 = (\pm ik)^2 \quad \Rightarrow \quad b = \pm ik
\]

**Two Solutions:**
\[
\psi_+(x) = Ae^{ikx} \quad \text{ADD TIME DEP.} \quad \Rightarrow \quad \psi_+(x,t) = Ae^{ikx-\omega t}
\]
\[
\psi_-(x) = Ae^{-ikx} \quad \text{ADD TIME DEP.} \quad \Rightarrow \quad \psi_-(x,t) = Ae^{-ikx+\omega t}
\]

**PROBABILITY DENSITY:**
\[
|\psi_\pm|^2 = \psi_\pm^*\psi_\pm = A^*Ae^{-\omega t} = A^*A = |A|^2 = \text{constant} \quad \Rightarrow \quad \text{everywhere equal!}
\]
FREE PARTICLE (CONT'D.)

- **Eigenfunctions?**
  - \( E \) \( \hat{H} \psi = E \psi \), so AUTOMATIC IN \( E \): EIGENVALUE OF
  \[
  E = \frac{-\hbar^2 k^2}{2m}
  \]
  But IF \( p = \frac{h}{\lambda} + k = \frac{\pi}{\lambda} \): \( p = \pm k \)
  
  AND
  \[
  E = \frac{p^2}{2m} \text{ (like classic.)}
  \]

- \( \hat{p} \psi = -i\hbar \frac{\partial \psi}{\partial x} = -i\hbar (\pm ik) \psi \)
  \[
  e^{\pm ikx} = (\pm ik)^j
  \]
  Two EIGENVALUES
  \[
  p_+ = \pm k \rightarrow
  p_+ \rightarrow\]
  \[
  p_- = -\pm k
  \]

- **Uncertainty Principle?**
  \[
  p_x = \pm p_\pm \text{ DEFINITE NO. } \pm \pm k \rightarrow \Delta x \Delta p_\pm \geq \frac{\hbar}{2}
  \]

- **Normalization?**
  \[
  \int_{-\infty}^{\infty} \psi^*_m \psi \, dx = \int |A|^2 \, dx = 1 \text{? NOT POSSIBLE, since UNUSUAL + INFINITE IN EXTENT.}
  \]

- **Correspondence Principle?**
  NOT OK: ANY \( E \)
  \[
  p_\pm \rightarrow 0 \rightarrow p_+
  \]
Quantum Mechanics in 1 Dimension

Particle in a rigid box:

\[ E_n = \frac{n^2 \pi^2 \hbar^2}{2mL^2} \]

Energy levels:
- \( E_1 = \frac{\pi^2 \hbar^2}{2mL^2} \)
- \( E_2 = \frac{4\pi^2 \hbar^2}{2mL^2} \)
- \( E_3 = \frac{9\pi^2 \hbar^2}{2mL^2} \)

Plus adding a variable barrier in the middle of the box:
http://www.chem.uci.edu/undergrad/applets/dwell/dwell.htm
 PARTICLE IN A FINITE BOX

\[ \Psi_\mu = C e^{i k x} + D e^{-i k x} \]
\[ \Psi_\nu = G \sin k x + D \cos k x; \quad k = \sqrt{\frac{2mE}{\hbar^2}} \]

\[ \Psi_I = A e^{\alpha x} \]
\[ \alpha = \sqrt{\frac{2m(U_0 - E)}{\hbar^2}} \]

DECAY LENGTH = \frac{1}{\alpha}

\[ \Psi_\lambda = B e^{-\alpha x} \]

\[ U = U_0 \]
\[ E < U_0 \text{ (BOUND)} \]
\[ U = 0 \]

CLASSICAL TURNING POINTS

CLASSICALLY FORBIDDEN = "TUNNELING"

Plus
http://www.chem.uci.edu/undergrad/applets/dwell/dwell.htm
Plus adding a variable barrier in the middle of the box:
http://www.chem.uci.edu/undergrad/applets/dwell/dwell.htm
QUANTUM HARMONIC OSCILLATOR:

$E_n = (n + \frac{1}{2})\hbar \omega$
$n = 0, 1, 2, \ldots$

Classical turning points:
$E_n = \frac{1}{2} k a_n^2$

$\alpha = \sqrt{\frac{m \omega}{\hbar}}$

Wave functions

$\psi_0(x) = \left( \frac{2}{\pi} \right)^{1/4} \frac{1}{\sqrt{\alpha}} e^{-\alpha x^2/2}$

$\psi_1(x) = \left( \frac{2}{\pi} \right)^{1/4} \frac{1}{\sqrt{2\alpha}} x e^{-\alpha x^2/2}$

$\psi_2(x) = \left( \frac{2}{\pi} \right)^{1/4} \frac{1}{\sqrt{6\alpha}} (2x^2 - 1) e^{-\alpha x^2/2}$

$\psi_3(x) = \left( \frac{2}{\pi} \right)^{1/4} \frac{1}{\sqrt{14\alpha}} (3x^3 - 3x) e^{-\alpha x^2/2}$

$\psi_n(x) = \left( \frac{2}{\pi} \right)^{1/4} \frac{1}{\sqrt{(2n+1)\alpha}} x^n e^{-\alpha x^2/2}$

$\psi_n(x) = \left[ \text{Hermite polynomial} \right] e^{-\alpha x^2/2}$

with:

$\alpha = \sqrt{\frac{m \omega}{\hbar}}$
QUANTUM HARMONIC OSC. & CORRESP. PRINC.

\[ |\psi_n|^2 \]

\[ \frac{1}{2} k a_n^2 = E_n \]

\( V = 0 \)

\( V = V_{\text{max}} \)

CLASSICAL RESIDENCE TIME

PROBABILITY

\( n = 0 \)

\( n = 1 \)

\( n = 2 \)

\( n = 3 \)

\( n = 10 \)

TUNNELLING
Expt.—The Photoelectric Effect on $H_2 \rightarrow H_2^+ + e^- +$ vibrations

$\Delta E = h\nu = (6.63 \times 10^{-34}) \times (6.961 \times 10^{13}) / (1.60 \times 10^{-19}) = 0.288 \text{ eV}$
Qualitative Forms of 1D Bound-State $\psi$'s:

For each "stairstep":

$U(x) > E$

$\psi(x) \propto e^{i k(x) x}$

$k(x) = \sqrt{2m(E-U(x)) / \hbar^2}$

$x(x) = \sqrt{2m(U(x)-E) / \hbar^2}$

$\lambda(x) = \frac{\hbar}{p(x)} = \frac{2\pi}{k(x)}$

$\lambda_{\text{II}} > \lambda_{\text{III}}$

$\lambda_{\text{III}} < \lambda_{\text{IV}}$

$|\psi(x)| = \text{max. near turning points}$

$|\psi(x)| = \text{min. near deepest part of } U(x)$

$\sim \propto 1/k(x) \propto 1/p(x)$

$U(x) \Rightarrow \text{piecewise constant "staircase"}$
In all cases, the overall time-dependent wave function is given by \( \Psi(x, t) = \psi(x) e^{iEt/\hbar} \), where \( E \) is the energy given in the table. All \( \psi(x) \)'s are also eigenfunctions of the Hamiltonian, since they satisfy \( \hat{H} \psi = -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} \psi(x) + U(x) \psi(x) = E \psi(x) \). In one dimension, \( \frac{\partial}{\partial x} \rightarrow \frac{d}{dx} \), and \( \frac{\partial^2}{\partial x^2} \rightarrow \frac{d^2}{dx^2} \).

<table>
<thead>
<tr>
<th>PROBLEM</th>
<th>HAMILTONIAN</th>
<th>WAVEFUNCTION ( \psi )</th>
<th>NORMALIZATION OF ( \psi )</th>
<th>ALLOWED ENERGIES ( E )</th>
<th>EIGENFUNCTION PROPERTIES (beyond ( \hat{H} ))</th>
<th>UNCERTAINTY PRINCIPLE: ( \Delta x \Delta p_x \geq \hbar/2 )</th>
<th>CORRESPONDENCE PRINCIPLE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Free particle: ( U(x) = 0 ) or a constant, ( \therefore F_x = 0 )</td>
<td>(-\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} + 0 )</td>
<td>( \psi(x) = e^{\pm ikx} ), +k is wave traveling to right, -k is wave traveling to left</td>
<td>Not possible in usual way, as ( \infty ) in extent</td>
<td>( E = ) any value</td>
<td>( \hat{p}_x \psi = -i\hbar \frac{\partial}{\partial x} \psi = (\pm \hbar k) \psi ), ( \because ) ( &lt;p_x&gt; = \pm \hbar k )</td>
<td>( \Delta p_x = 0 ), since ( \psi ) is eigenfunction, ( \therefore \Delta x = \infty )</td>
<td>Special: equal probability of finding particle anywhere in x</td>
</tr>
<tr>
<td>Particle inside a rigid box</td>
<td>(-\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} + 0, ) inside box (-\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} + \infty, ) outside box</td>
<td>( \psi_n(x) = \sqrt{\frac{2}{L}} \sin \left( \frac{nx}{L} \right) ) ( = ) standing wave inside box, ( \psi_n(x) = 0 ), outside box</td>
<td>Usual, by requiring ( \int \psi^*(x) \psi(x) dx = 1 )</td>
<td>( E_n = n^2 \frac{\pi^2 \hbar^2}{2mL^2} ) None, but if box centered on ( x=0 ): then all ( \psi )’s even or odd, and parity ( \hat{\Gamma} ) gives: ( \hat{\Gamma} \psi(x) = \psi(-x) = \pm \psi(x) )</td>
<td>( \Delta x \approx L, \therefore \Delta p_x \geq h/(2L) )</td>
<td>For large n, equal probability of finding particle anywhere in box</td>
<td></td>
</tr>
<tr>
<td>Particle inside a soft box</td>
<td>(-\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} + 0, ) inside box (-\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} + U_0, ) outside box</td>
<td>( \psi_n(x) \propto \sin kx ) or ( \cos kx ), inside box ( \psi_n(x) \propto e^{kx} ) for negative ( x ), ( \psi_n(x) \propto e^{-kx} ) for positive ( x )</td>
<td>Usual</td>
<td>( E_n = ) quantized values, lower than those of same ( n ) for rigid box</td>
<td>All even or odd ( \psi )’s if box centered at zero, so eigenfunctions of parity ( \Delta x ) larger than for rigid box with same ( n ), ( \therefore \Delta p_x ) smaller</td>
<td>If ( U_0 ) large enough, like rigid box</td>
<td></td>
</tr>
<tr>
<td>Harmonic oscillator: ( F_x = -\kappa x )</td>
<td>(-\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} + ) ( \frac{1}{2} \kappa x^2 )</td>
<td>( \psi_n(x) \propto (n^{th}) \text{ order Hermite polynomial in } x) e^{-\alpha x^2} )</td>
<td>Usual</td>
<td>( E_n = (n + \frac{1}{2}) \hbar \omega_{\text{class}} )</td>
<td>All even or odd ( \psi )’s since potential centered at zero, so eigenfunctions of parity</td>
<td>Special for ground state ( \psi_n, ) since ( \Delta x \Delta p_x = \hbar/2 )</td>
<td>For large ( n ), converges to classical oscillator probability</td>
</tr>
</tbody>
</table>
Quantum Mechanics in 3 Dimensions: Separation of Variables...again

Degeneracy:
When >1 unique QM state has the same energy.
If \( L_x = L_y = L_z \):

\[
\text{Energy} \quad n_x \quad n_y \quad n_z \quad \text{Degen.}
\]

\[
E_0 = \frac{3\pi^2 \hbar^2}{2mL^2}
\]

\[
E_1 = 2E_0
\]

\[
E = E_x + E_y + E_z = \frac{\pi^2 \hbar^2}{2m} \left[ \frac{n_x^2}{L_x^2} + \frac{n_y^2}{L_y^2} + \frac{n_z^2}{L_z^2} \right]; \quad n_x = 1, 2, \ldots
\]

\[
n_y = 1, 2, \ldots
\]

\[
n_z = 1, 2, \ldots
\]
TUNNELING:

\[ T = \frac{F^* F}{A^* A} = \frac{|F|^2}{|A|^2} \]

- For \( E = U_0 \):
  \[ T = \left( 1 + \frac{U_0^2 \sin^2(kL)}{4E(U_0 - E)} \right)^{-1} \]

- For \( KL \ll 1 \):
  \[ T \approx U_0 \left( 1 - \frac{E}{U_0} \right) e^{-2kL} \]

- For \( E > U_0 \):
  \[ T = \left( 1 + \frac{U_0^2 \sin^2(k_{\text{II}}L)}{4E(U_0 - E)} \right)^{-1} \]

**Classical**

\[ \frac{\hbar^2}{2} = \frac{\pi}{2} \frac{2\pi}{k_{\text{II}}} \]

**Resonances** when \( L = \frac{\pi}{2} \frac{2\pi}{k_{\text{II}}} \)

\[ \psi_{\text{II}} = C e^{-ik_{\text{II}}x} + D e^{ik_{\text{II}}x} \quad (E > U_0) \]

\[ \psi_{\text{III}} = C e^{-ik_{\text{III}}x} + D e^{ik_{\text{III}}x} \quad (E < U_0) \]

\[ k_{\text{II}} = \sqrt{\frac{2mE}{\hbar^2}} \]

\[ k_{\text{III}} = \sqrt{\frac{2mE}{\hbar^2}} \]

Dual-De Broglie wavelengths:

\[ \frac{\hbar^2}{2} = \frac{\pi}{2} \frac{2\pi}{k_{\text{II}}} \]

\[ \frac{\hbar^2}{2} = \frac{\pi}{2} \frac{2\pi}{k_{\text{III}}} \]
Tunnel Effect, Stationary States

$$\text{Re}\phi(E,x)$$

$u = u_0$

$u = 0$

$E < u_0$

$E = u_0$

$E > u_0$

$$\lambda_1 = \lambda_III < \lambda_II$$
TUNNELING OF A WAVE PACKET

http://www.youtube.com/watch?v=2WEXTyCAQ18&NR=1
Fig. 18  Gaussian wave-packet scattering from a square well when the mean energy of the packet is half the well depth.
TUNNELING AND ALPHA DECAY IN NUCLEI

Nucleus (+Ze)
Alpha particle (+2e)

\[ \sim 10^{21} \text{ collisions/s} \approx \frac{V_{\text{coll}}}{2kR} = f_{\text{coll}} \]

\[ E_{\alpha} = K_{\alpha} = \frac{1}{2} m v_{\alpha}^2 = 5 \text{ MeV} \]

\[ R_0 \approx 20 \times 10^{-15} \text{ m} \]

(+ See Ex. 6.17)

\[ U(r) = \frac{2kZe^2/r}{r} = \text{repulsive Coulomb} \]

\[ U(r) = 2kZe^2/r \]

\[ R = 2kZe^2/E \]

\[ T(E) \approx \exp\left\{ -4\pi Ze^2 \left[ \frac{E}{E_{\text{ex}}} + 8 \frac{Zr_0}{E_{\text{ex}}} \right] \right\} \]

\[ \lambda = f_{\text{coll}} \frac{T(E)}{\alpha} \text{ (No./sec-nucleus)} \]

\[ 0.0993 \text{ MeV} \]

\[ 7.3 \times 10^{-15} \text{ m} \]
OPERATION OF THE TUNNEL DIODE

Very fast switching for computer applications
FIELD EMISSION FROM A METAL

\[ e^- \text{ with energy } E < 0 \]

Electric Field

\[ \varepsilon = \]

\[ U(x) = -e\varepsilon x \]

WORK FUNCTION: \( \phi \approx q \approx 4\text{eV} \)

\[ x_1 = 0 \]

\[ x_2 = -\frac{E}{e\varepsilon} \]

\[ \varepsilon \approx 10^6 \text{V/mm} \]

\[ \text{NO VOLTAGE} \]

\[ \text{Axis normal to surface} \]

\[ \text{NO. } e^-/\text{SEC.} \]

\[ \approx 10^{30}, 10^{-20} \]

\[ \approx 10^{10} \approx 1 \text{nA} \]

\[ \text{CLASSIC. FORBIDDEN} \]

\[ \approx 10^{30} \text{ COLLISIONS} \]

\[ \text{PER SEC} \]

\[ T(E = \phi) = \exp \left( \left( \frac{4}{3} m_1 \Omega \left( \frac{3}{2} \right)^{3/2} \frac{1}{E} \right) \right) \approx 10^{-20} \]
Figure 4 Scanning tunneling microscopes can be operated in either (a) the constant current mode or (b) the constant height mode. The images of the surface of graphite were made by Richard Sonnenfeld at the University of California at Santa Barbara. The constant height mode was first used by A. Bryant, D. P. E. Smith, and C. F. Quate, Applied Physics Letters 48: 832, 1986.
Figure 3  (a) The wavefunction of an electron in the surface of the material to be studied. The wavefunction extends beyond the surface into the empty region. (b) The sharp tip of a conducting probe is brought close to the surface. The wavefunction of a surface electron penetrates into the tip, so that the electron can "tunnel" from surface to tip. Compare this figure to Figure 6.7b.

\[ j = \text{current/} \text{unit area} \]

\[ \approx \frac{e^2 \Delta V}{4\pi^2 L \varepsilon_0} e^{-2L/\delta} \propto e^{-2 \xi L} \]

\[ \delta = \frac{1}{\xi} = \sqrt{\frac{\hbar^2}{2m(U-E)}} \approx 1.0 \text{Å} \]
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