Physics 9HD, Midterm 1, October 27, 2006

Closed book test. No books or notes. You may use your calculator for arithmetic only. SHOW YOUR WORK FOR PARTIAL CREDIT. ANSWERS ALONE WILL NOT RECEIVE CREDIT! Total 100 points. Note that the easier problems, which are worth fewer points, are first.

1. (5 pts) Suppose a point charge $Q$ is located inside a spherical cavity of radius $A$ inside a cylindrical solid conductor of radius $B$ and length $L$, where $A < B$. The cylindrical conductor has a total charge of $10Q$ on its inner and outer surfaces. Find the charge on the inner and outer surfaces of the cylindrical conductor.

2. (10 pts) Suppose that a point charge $Q_A = 3\text{esu}$ is located at the origin of a Cartesian coordinate system. A second point charge $Q_B = 5\text{esu}$ is located on the x-axis at $x = 2 \text{ cm}$. Find the electric field on the x-axis at $x = 5 \text{ cm}$. Then use that field to find the force on a third point charge, $Q_C = -2 \text{ esu}$, which is placed on the x-axis at $x = 5 \text{ cm}$.

3. (10 pts) If the potential is $\varphi = 15xy + 3y^2 + 5z^2$, find the electric field $\vec{E}(x, y, z)$. Explicitly calculate the curl of $\vec{E}$ and prove that it is zero, as expected for an electrostatic field.

4. (15 pts) Suppose there is a uniform, linear charge distribution of $\lambda = -\frac{5\text{esu}}{cm}$ on a one-dimensional wire of length 10 cm which is on the x-axis, located between origin and $x = 10 \text{ cm}$. Find the electric field at the point on the x-axis where $x = 15 \text{ cm}$.

5. (20 pts) Consider a coaxial cable, which has a solid inner conductor of radius $a$, an outer cylindrical conductor of radius $b$, and a length $L$, where $L >> b, a$.
   (a) Use Gauss’s Law to derive the $\vec{E}$ field between the two conductors, if there is a linear charge density of $+\lambda$ on the inner conductor and $-\lambda$ on the outer conductor.
   (b) Now find the capacitance of the cable.
   (c) Find the energy stored in the field between the two conductors, and show that this energy is of the form $\frac{Q^2}{2C}$, where $C$ is the capacitance of the cable.

6. (40 pts) Consider a spherical charge distribution with radius $a$ and volume charge density $\rho(r) = \rho_o r$, where $\rho_o$ is a positive constant and $r$ is the distance from the center of the sphere.
   (a) Find the electric field and the potential, both outside and inside the spherical charge distribution. Assume that potential is zero at infinity. [Hint: First find the electric field and potential outside the spherical charge distribution. When you find the potential inside the spherical charge distribution, note that the potential is a continuous function of $r$ for all values of $r$, i.e., $\varphi(r)$ is continuous at $r = a$.]
   (b) Suppose an electron is released from rest at infinity and is attracted to the spherical charge distribution. What is the kinetic energy of the electron when it reaches the surface of the sphere?