Physics 9HD, Midterm 2 review
DC Circuits

Current \( I = \frac{dq}{dt} \)

Ohm’s Law, \( V = IR \) for resistor, \( \vec{J} = \sigma \vec{E} \) where \( \sigma \) is electrical conductivity=1/\( \rho \), and \( \vec{J} \) is the current density, i.e., \( I = \int \vec{J} \cdot d\vec{a} \).

\( \vec{J} = ne\vec{v} \), where \( n= \) number of carriers/volume, \( e= \) charge of carriers, and \( \vec{v} \) is the average velocity of the carriers.

Resistance of wire in terms of resistivity: \( R = \frac{\rho L}{A} \), length \( L \), area \( A \)

Battery \( \mathcal{E} \) with internal resistance \( r \) develops effective voltage \( \Delta V = \mathcal{E} - Ir \)

Kirchhoff’s Laws:
1. Current Law: Sum of currents entering a node = sum of currents leaving that node. (Charge conservation)
2. Voltage Law: Sum of voltage drops around a closed loop is zero. (Energy conservation)

Series Circuit has same current in each element
Series resistances: \( R_{eq} = R_1 + R_2 + R_3 + \ldots + R_n \)

Parallel Circuit has same voltage across each element
Parallel resistances: \( \frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots + \frac{1}{R_n} \)

Power dissipated in circuit: \( P = IV \)

Power dissipated in resistor: \( P = I^2R = \frac{V^2}{R} \)

Charging capacitor with battery \( \mathcal{E} \), \( q = CE(1 - e^{-\frac{t}{RC}}) \); time constant \( \tau = RC \)

Discharging capacitor: \( q = Q_0 e^{-\frac{t}{RC}} \)

Thevenin equivalent circuit: \( \mathcal{E}_{eq} = \) open circuit voltage, \( I_{sc} = \) short circuit current, \( R_{eq} = \mathcal{E}_{eq} / I_{sc} \)

Lorentz Force on Charge in Electric and Magnetic Fields
\( \vec{F} = q\vec{E} + q\frac{\vec{v}}{c} \times \vec{B} \)

Motion of charge in constant magnetic field is circle or helix
Cyclotron motion, \( \vec{v} \perp \vec{B} \), \( m\vec{v}=Bq\vec{r} \), where \( r \) is the radius of the circle; derived by setting

\[
\text{centripetal force equal to magnetic force} \quad |\vec{F}| = \frac{mv^2}{r} = q \frac{v}{c} B
\]

Magnetic field does no work, because \( W = \int \vec{F} \cdot d\vec{l} = 0 \), since \( \vec{F} \perp \vec{v} \Rightarrow \vec{F} \perp d\vec{l} \).

Velocity selector: \( \vec{v} \perp \vec{E}, \vec{E} \perp \vec{B} \), Electric force = Magnetic force, i.e., \( qE=qvB/c \Rightarrow v=E/cB \).

For current carrying conductor, \( \vec{F} = \frac{I\vec{I} \times \vec{B}}{c} \), \( d\vec{F} = \frac{I d\vec{l} \times \vec{B}}{c} \).
**Magnetic Fields**
\[ \oint B \cdot d\mathbf{a} = 0, \quad \nabla \cdot \mathbf{B} = 0, \] so magnetic field lines always form closed loops (no magnetic monopoles)

Hall effect: \[ \vec{E}_i = \frac{-\vec{j} \times \vec{B}}{nqc}, \] sign of \( \vec{E} \), depends on sign of charge carriers \( q \).

Biot-Savart Law: \[ d\vec{B} = \frac{\vec{I} \times \mathbf{r}}{cr^2}. \] Know how to calculate \( \vec{B} \) field of straight current-carrying conductor, field at center of current loop, field on axis of current loop.

Ampere’s Law: \[ \oint \vec{B} \cdot d\mathbf{l} = \frac{4\pi}{c} I_{\text{enclosed}} = \frac{4\pi}{c} \int \vec{j} \cdot d\mathbf{a} \]

Use of Ampere’s Law to find magnetic field of infinitely long wire \( \vec{B} = \frac{2I}{cr} \hat{\phi} \) carrying current \( I \).

Know how to find magnetic field of long wire with current density \( \vec{J}(r) \), which depends on radial cylindrical coordinate \( r \).

- Solenoid: \( \vec{B} = \frac{4\pi n I}{c} \), where \( n \)=#turns/length
- Torus: \( \vec{B} = \frac{2NI}{cr} \), where \( r \) is a radius inside the doughnut, \( N \)=number of turns

Force between parallel wires:
- Wires attract if current is in same direction. Wires repel if current is in opposite directions.
\[ F = \frac{2LI_1L}{c^2r} \]

**Special Relativity**
\[ \beta = \frac{v}{c}, \quad \gamma = \frac{1}{\sqrt{1 - \beta^2}} \]

Charge is invariant under Lorentz transformation
\( (ct, \vec{x}) \) is a 4-vector, as is \( \left( \frac{c}{\gamma} E, \gamma \vec{p} \right) \)

Lorentz transformation for \( (ct, \vec{x}) \) (time and space 4-vector), with frame \( F' \) moving in +x direction with speed \( v \) as viewed from frame \( F \):
\[ ct' = \gamma (ct - \beta x) \]
\[ x' = \gamma (x - \beta ct) \]
\[ y' = y \]
\[ z' = z \]

Length contraction \( \Delta x = \frac{1}{\gamma} \Delta x' \), rest length of object measured in \( F' \), moving objects are shortened.

Time dilation \( \Delta t = \gamma \Delta t' \), moving clock in \( F' \) runs slow, time measured in other frames is longer.

Addition of velocities: \[ \beta' = \frac{\beta - \beta_o}{1 - \beta \beta_o} \]

Transformation of forces: \[ f'_\parallel = f_\parallel, \quad f'_\perp = \frac{f_\perp}{\gamma} \]
Transformation of electric field: $E_{\parallel}' = E_{\parallel}, \ E_{\perp}' = \gamma E_{\perp}$

Electric field of moving charge $Q$: $\vec{E}' = \frac{Q}{\gamma^2 r'^2 (1 - \beta^2 \sin^2 \theta')}$, where $\theta'$ is the angle between a vector $\vec{r}'$ to the observation point and the velocity vector $\vec{v}$ of the particle.

Electric and magnetic fields are interrelated in Lorentz transformation. Here is how we derived the existence of a magnetic field, using Lorentz transformations:

Consider a neutral wire in lab frame ($E=0$), with a net current in (-x) direction in the wire. There is no electric force on a moving test charge in lab frame, but there is a net force on it. We can find this net force by transforming to the rest frame of the test charge. There we will find an $E'$ field, which exerts a force on the test charge. When we transform that force back to the lab frame, there must also be a force on the test charge in the lab frame. We can interpret the source of the force in the lab frame as due to a magnetic field $\vec{B} = \frac{2I}{r_c} \hat{z}$ caused by the current $I$, and exerting a force on the test charge of $\vec{F} = \frac{q}{c} \vec{v} \times \vec{B}$.