Electric Field of Point Charge Moving with Constant Velocity

In frame F, charge Q is at rest at the origin. \( \mathbf{E} = \frac{Q}{r^2} \)

In the x'z plane, the fields are:

\[
E_x = \frac{Q \cos \theta}{r^2} \left( \frac{x'}{r^2} + z^2 \right)^{3/2} \\
E_z' = \frac{Q \sin \theta}{r^2} \left( \frac{z'}{r^2} + z^2 \right)^{3/2}
\]

\( F' \) moves in -x direction with speed v, with respect to F.

Then Q moves in +x' direction with speed v in F'.

\( x = y(x' - \beta ct'), \ y = y', \ z = z', \ ct = \gamma (ct' - \beta x') \)

At \( t' = 0 \), when Q passes the origin in F',

\[
E_x' = \frac{Q x'}{\left( (y')^2 + z'^2 \right)^{3/2}} \\
E_z' = \frac{Q z'}{\left( (y')^2 + z'^2 \right)^{3/2}}
\]

Note \( \frac{E_z'}{E_x'} = \frac{z'}{x'} \), i.e. \( \mathbf{E}' \) makes same angle with x' axis as does \( r' \).

\( \mathbf{E}' \) points radially outward along a line drawn from instantaneous position of Q.

\[ E'^2 = E_x'^2 + E_z'^2 = \frac{y^2 Q^2 (x'^2 + z'^2)}{\left( (y')^2 + z'^2 \right)^3} \]

\[ = \frac{Q^2 (x'^2 + z'^2)}{y^2 \left[ x'^2 + (1 - \beta^2) z'^2 \right]^3} \]

\[ = \frac{Q^2}{\left[ x'^2 + (1 - \beta^2) z'^2 \right]^3} \]

\[ = \frac{Q^2}{\left[ x'^2 + (1 - \beta^2) \sin^2 \theta' \right]^3} \]

\[ \|
\mathbf{E}' \| = \frac{Q}{\gamma^2 r'^2 (1 - \beta^2 \sin^2 \theta')^{3/2}} \]

\[ \Rightarrow \frac{Q}{\gamma^2 r^2 (1 - \beta^2 \sin^2 \theta)^{3/2}} \]

\[ \Rightarrow \frac{Q}{\rho^2} \]

For large \( \beta \) (near 1), \( \mathbf{E} \) is longer by factor \( \gamma \) in + direction \( \frac{x'}{\rho} \)

and short by \( \gamma^2 \) in \( \theta ' \) direction

No stationary charge distribution could have these field lines.

Note \( \int E' \cdot d\mathbf{s}' = 0 \)