Chapter 10

\( \triangle \geq 3. \) In the ground state \( E = (n + \frac{1}{2}) \hbar \omega - \frac{1}{2} \hbar \omega - \frac{1}{2} \hbar \omega = \frac{1}{2} \hbar \omega - \frac{1}{2} \hbar \omega - \frac{1}{2} \hbar \omega \). Solving for \( \lambda \):

\[
\lambda = \sqrt{\frac{\hbar}{\mu \omega}}
\]

Using the \( ^{35} \text{Cl} \) isotope

\[
\mu = m_1 m_2 = \frac{35}{36} m_1 m_2 = 1.614 \times 10^{-27} \text{ kg}
\]

From Table 10.1 \( f = 8.66 \times 10^{13} \) Hz. With \( \omega = 2\pi f \) we have

\[
A = \sqrt{\frac{\hbar}{\mu \omega}} = \sqrt{\frac{1.055 \times 10^{-34} \text{ J} \cdot \text{s}}{(1.614 \times 10^{-27} \text{ kg})(2\pi)(8.66 \times 10^{13} \text{ s}^{-1})}} = 1.10 \times 10^{-11} \text{ m}
\]

\( \Delta E = E_1 - E_0 = E_1 = \frac{\hbar^2}{I} = \frac{\hbar \lambda}{\lambda} \)

\[
I = \frac{\hbar^2 \lambda}{2\pi \hbar c} = \frac{(1.055 \times 10^{-34} \text{ J} \cdot \text{s})(1.3 \times 10^{-3} \text{ m})}{2\pi(2.998 \times 10^8 \text{ m/s})} = 7.28 \times 10^{-47} \text{ kg} \cdot \text{m}^2
\]

b) The minimum energy in a vibrational transition is \( \Delta E = hf \). From Table 10.1 \( f = 6.42 \times 10^{13} \) Hz, which corresponds to a photon of wavelength \( \lambda = c/f = 4.67 \mu \text{m} \). A photon of this wavelength or less is required to excite the vibrational mode, so the 1.30 mm photon is too weak.

\( \leq \star 18. \) a) Using dimensional analysis and the fact that the energy of each photon is \( \hbar c/\lambda = 3.14 \times 10^{-19} \text{ J} \),

\[
N = \frac{5 \times 10^{-3} \text{ J/s}}{3.14 \times 10^{-19} \text{ J/photon}} = 1.59 \times 10^{16} \text{ photon/s}
\]

b) 0.02 mole is equal to \( 0.02 N_A = 1.20 \times 10^{22} \) atoms. Then the fraction participating is

\[
\frac{1.59 \times 10^{16}}{1.20 \times 10^{22}} = 1.33 \times 10^{-6}
\]

c) The transitions involved have a fairly low probability, even with stimulated emission. We are saved by the large number of atoms available.

\( \leq \) 21. a) From Chapter 9 we have

\[
\Delta f = \frac{f_0}{c} \sqrt{\frac{kT}{m}}
\]

We also have in general \( f = c/\lambda \) so \( \Delta f = (c/\lambda^2) \Delta \lambda \). Therefore

\[
\Delta \lambda = \frac{\lambda^2 \Delta f}{c} = \frac{\lambda^2 f_0}{c^2} \sqrt{\frac{kT}{m}} = \frac{\lambda}{c} \sqrt{\frac{kT}{m}}
\]

Using the neon mass \( 3.32 \times 10^{-26} \text{ kg} \) we calculate

\[
\Delta \lambda = \frac{6.328 \times 10^{-7} \text{ m}}{2.998 \times 10^8 \text{ m/s}} \sqrt{\frac{(1.381 \times 10^{-23} \text{ J/K})(293 \text{ K})}{3.32 \times 10^{-26} \text{ kg}}} = 7.37 \times 10^{-13} \text{ m}
\]

b)

\[
\Delta \lambda = \frac{\hbar \lambda^2}{2 \hbar c} \Delta \lambda = \frac{\lambda^2}{4 \pi \sigma c} \Delta \lambda = \frac{(6.328 \times 10^{-7} \text{ m})^2}{4 \pi (2.998 \times 10^8 \text{ m/s})(10^{-3} \text{ s})} = 1.06 \times 10^{-19} \text{ m}
\]

The Doppler broadening is much more significant than the Heisenberg broadening.
\[ \Delta t = \frac{2(1 \text{ m})}{2.998 \times 10^8 \text{ m/s}} = 6.67 \times 10^{-9} \text{ s} \]

b) For the 16 km round trip

\[ \Delta t = \frac{16 \times 10^2 \text{ m}}{2.998 \times 10^8 \text{ m/s}} - \frac{16 \times 10^3 \text{ m}}{(1 - 3 \times 10^{-4}) \times 2.998 \times 10^8 \text{ m/s}} = -1.6 \times 10^{-8} \text{ s} \]

Because this result is larger than the desired uncertainty in timing, it is important to take atmospheric effects into account.