Physics 9D-Modern Physics
Midterm, Version 1
4 May, 2006
(100 points total)
You may tear this sheet off.

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Miscellaneous data and equations:

c = 3.00 \times 10^8 \text{ m/s} \quad e = 1.60 \times 10^{-19} \text{ C} \quad 1 \text{ eV} = 1.60 \times 10^{-19} \text{ J} \quad 1 \text{ Å} = 10^{-10} \text{ m}

M_{\text{Sun}} = 2 \times 10^{30} \text{ kg} \quad M_{\text{Earth}} = 5.98 \times 10^{24} \text{ kg} \quad r_{\text{Earth}} = 6.38 \times 10^6 \text{ m}

m_e = 9.1094 \times 10^{-31} \text{ kg} \quad 0.5110 \text{ MeV/c}^2 \quad m_p = 1.6726 \times 10^{-27} \text{ kg} \quad 938.27 \text{ MeV/c}^2

m_n = 1.6749 \times 10^{-27} \text{ kg} \quad 939.57 \text{ MeV/c}^2 \quad 1 \text{ u} = 1.6605 \times 10^{-27} \text{ kg} \quad 931.49 \text{ MeV/c}^2

m(^1\text{H}) = 1.0078 \text{ u} \quad G = 6.67 \times 10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2 \quad g = 9.81 \text{ m/s}^2 \quad \sigma = 5.67 \times 10^{-8} \text{ W}\cdot\text{m}^{-2}\cdot\text{K}^{-4}

\hbar = 6.63 \times 10^{-34} \text{ J-s} \quad \hbar = h/2\pi \quad k_B = 1.38 \times 10^{-23} \text{ J}\cdot\text{K}^{-1} \quad a_0 = 0.529 \text{ Å}

N(t) = N_0 \exp(-t/\tau_0) = N_0 \exp(-0.693t/\tau^{1/2}) \quad k_C = 1/(4\pi\varepsilon_0) = 8.98 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2 \quad R_H = 1.09678 \times 10^7 \text{ m}^{-1}

\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = \sqrt{1+0.5\beta^2} \quad \text{if} \quad v << c \quad \beta = v/c = [(\gamma^2-1)\gamma^2]^{1/2}

\frac{T}{T_0} = \gamma \quad L = L_0\gamma \quad \nu = \nu_0 \left(1 \pm \beta\right)^{1/2} \quad \equiv \nu_0 [1 \pm \beta] \quad \text{for} \quad \beta << 1 \quad \nu = c/\lambda \quad \nu \text{ (lect.)} = f(\text{book})

x' = \gamma(x-vt) \quad y' = y \quad z' = z \quad t' = \gamma [t - \left(\frac{v}{c^2}\right)x]

u'_x = \frac{u_x - v}{\sqrt{1 - \left(\frac{v}{c}\right)^2}u_x} \quad u'_y = \frac{u_y}{\gamma \sqrt{1 - \left(\frac{v}{c}\right)^2}u_x} \quad u'_z = \frac{u_z}{\gamma \sqrt{1 - \left(\frac{v}{c}\right)^2}u_x}

x'^2 + y'^2 + z'^2 - c^2t'^2 = \text{invariant}

\beta = \gamma m \mu \quad E = \gamma mc^2 \quad K + mc^2 \quad E^2 = p^2c^2 + m^2c^4

E = pc = \frac{\hbar \nu}{c} \quad \frac{\Delta v}{\nu} = \frac{\Delta T}{T} = -\frac{GM}{c^2} \left[\frac{1}{r_1} - \frac{1}{r_2}\right] \approx -\frac{gH}{c^2} \quad \frac{\Delta v}{\nu} \approx -\frac{\Delta \lambda}{\lambda} \quad \text{if} \quad \frac{\Delta v}{\nu} \ll 1 \quad R_{\text{sch}} = \frac{2GM}{c^2}

\frac{e}{m_e} = \frac{V \tan \theta}{dB^2} \quad \lambda_{\text{max}} = 6.289 \times 10^{-3} \text{ m-K} \quad l(\lambda, T) = \frac{2\pi c^2 h}{\lambda^5} \cdot \frac{1}{e^{\frac{2\pi c^2 h}{\lambda k T}} - 1} \quad R(T) = 8\sigma T^4

N(\theta) = \frac{N_{nt}}{16} \left(\frac{e^2}{4\pi\varepsilon_0}\right)^2 \frac{Z_1^2 Z_2^2}{r^4 K^2 \sin^4(\theta/2)} \quad b = \frac{Z_1 Z_2 e^2}{8\pi\varepsilon_0 K} \cot(\theta/2) \quad r_{\min} = \frac{Z_1 Z_2 e^2}{4\pi\varepsilon_0 K}

f(\geq \theta) = nt \sigma = nt \pi b^2
\[ h\nu = K_{\text{max}} + \phi \quad \lambda_{\text{min}} = \frac{hc}{eV} \quad n\lambda = 2\text{dsin}\theta \]

\[ E_n = -\frac{Ze^2}{8\pi\varepsilon_0 a_n n^2} = -\frac{13.6Z}{n^2} \text{ (eV)} \quad r_n = \frac{4\pi\varepsilon_0 h^2}{m_e Ze^2} n^2 = a_0 \frac{n^2}{Z} \]

\[ \nu_n = \frac{nh}{m_e r_n} \quad \frac{1}{\lambda} = Z R_n \left[ \frac{1}{n_l^2} - \frac{1}{n_u^2} \right] \quad \mu_e = m_e \left[ \frac{M}{M + m_e} \right] \quad \lambda = h/p \]

\[ \Delta\lambda = \lambda' - \lambda = \frac{h}{m_e c} \left[ 1 - \cos \theta \right] = 2.426 \times 10^{-9} \left[ 1 - \cos \theta \right] \text{ (in m)} \]
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[1] (40 Points) Relativity

(a) The two Mars rovers that recently landed there traveled a distance of 800 million kilometers in let us say exactly 7.000 months as measured by a clock on the spacecraft. To an observer on earth, how many months did the journey take?

First, let's rewrite the speed of light in terms of miles and months:

\[ c = \left( 3 \times 10^8 \frac{m}{s} \right) \left( \frac{1 \text{ mi}}{1610 \text{ m}} \right) \left( \frac{86400 \text{ s}}{1 \text{ day}} \right) \left( \frac{30 \text{ days}}{1 \text{ month}} \right) = 4.83 \times 10^{11} \frac{\text{mi}}{\text{mo}} \]

Speed of the spacecraft:

\[ v = \frac{d}{t} = \frac{800 \times 10^6 \text{ mi}}{7 \text{ mo}} = 1.14 \times 10^7 \frac{\text{mi}}{\text{mo}} \]

Since \( v \ll c \), we have:

\[ \Delta t = \gamma t_0 \approx t_0 \left( 1 + \frac{v^2}{c^2} \right) \]

\[ \Delta t \approx 7 \text{ mo} \left( 1 + \frac{1.14 \times 10^7}{4.83 \times 10^{11}} \right) = 7,000,000.2 \text{ months} \]

---Choose one of the two parts (b) and (b') only: same credit for each---

(b) An observer in the future standing on Mars sees two spacecraft, one from Europe and one from the US, coming from opposite directions at speeds of 0.8c and 0.9c, respectively. What does someone on the European spacecraft measure for the speed of approach to Mars? And, for the European observer, what is the speed of approach of the US spacecraft?

An observer on the European ship sees the moon approach at 0.8c. From our rule for relativistic velocity addition, the velocity of the US ship according to the European observer is:

\[ U_x = \frac{U_x'}{1 - \frac{U_x U_y}{c^2}} = \frac{0.8c - (0.9c)}{1 + (0.8)(0.9)} = \frac{1.70}{1.72} c = 0.988 c \]
(b') Twin A makes a round trip from earth to a star that is 10 light-years away at a speed of 0.5c, while twin B stays on the earth. Each twin sends the other a signal once a year as measured by a clock at rest with each one. How many signals does A send during the trip? How many does B send? How many does A receive? How many does B receive?

The round trip time measured by B is: \( \Delta t = 2 \left( \frac{10 \text{ ly}}{0.5c} \right) = 40 \text{ years} \).

Thus, B sends out 40 signals.

B measures dilated time: \( \Delta t = \sqrt{1 - \left( \frac{0.5c}{c} \right)^2} = 1.15 \Delta t' \)

Thus, B receives \( \frac{40}{1.15} \approx 34 \) signals.

The round trip distance, perceived by A is: \( d_A = \frac{L}{\Delta t} = \frac{10 \text{ ly}}{1.15} = 8.7 \text{ ly} \).

The time A perceives is: \( \Delta t_A = d_A \Delta t' = 2 \frac{8.7 \text{ ly}}{0.5c} = 34.8 \text{ years} \).

Thus, A sends out 34 signals, but due to time dilation, A receives 40 signals.

(c) Light is emitted at a frequency of \( 7 \times 10^{14} \text{ Hz} \) from a source sitting atop the Eiffel Tower in Paris, which is 321 m in height. What would be the fractional change in the frequency of this light as observed on the ground and in which direction would it change (increase or decrease)?

\[
\frac{\Delta v}{v} = \frac{c^2}{v^2} = \frac{(9.8 \text{ m/s}^2)(321 \text{ m})}{(3 \times 10^8 \text{ m/s})^2} \approx 3 \times 10^{-4} \text{ faster}
\]
(d) What is the minimum mass (in solar masses) of a star with the density of nuclear matter that collapses to a black hole? Nuclear matter has a density of approximately $2.3 \times 10^{17} \text{ kg/m}^3$, the mass of the sun is $2.0 \times 10^{30} \text{ kg}$, and $V_{\text{sphere}} = \frac{(4/3)\pi r^3}{3}$. 

\[ r_S = \frac{2GM}{c^2} \quad \text{and} \quad \rho = \frac{M}{V} = \frac{3M}{\frac{4\pi}{3} r^3} \quad \text{let} \quad r = r_S \]

\[ r = \frac{2GM}{c^2} \Rightarrow r^3 = \frac{8G^3M^3}{c^6} \quad \text{then} \quad \rho = \frac{3c^6}{32\pi G^2 M^2} \]

Solving for $M$:

\[ M = \sqrt{\frac{3c^6}{32\pi G^2 \rho}} = \left[ \frac{3(3\times10^6 \text{ m/s})^6}{32\pi \left(6.67 \times 10^{-11} \text{ m}^3 / \text{kg} \cdot \text{s}^2\right) (2.3 \times 10^{17} \text{ kg})} \right]^{1/2} = 1.78 \times 10^3 \text{ kg} = 8.93 M_\odot \]

[2] (60 Points) Miscellaneous topics. Answer the following with brief, clear replies:

(a) Concisely state the two postulates of Special Relativity.

1. Principle of Relativity: The laws of physics are the same in all inertial frames.

2. The speed of light is the same in all inertial frames.

(b) What are the three forces of relevance in the Millikan oil drop experiment?

1. The gravitational force pulling the drop down.

2. The friction caused by air resistance, which is always opposite to the direction of motion.

3. The electrostatic force which is controlled by the experimenter, and is in either the upward or downward direction.
(c) What are the two distinct ways that x-rays can be produced?

1. Bremsstrahlung - the inverse photoelectric effect

2. Electron transitions between inner atomic orbitals

(d) At what frequency would radiation be emitted if a hydrogen atom made a transition from the n = 10 to n = 3 Bohr orbit?

Recall $v = \frac{\lambda}{\Delta}$ and substitute into equation 4.30:

$v = c \frac{R_{\infty}}{n^2} \left( \frac{1}{n_2^2} - \frac{1}{n_1^2} \right)$

$v = (3 \times 10^8 \text{ m/s}) (1.697 \times 10^{-7} \text{ m}) \left( \frac{1}{9} - \frac{1}{100} \right) = \frac{3.32 \times 10^4 \text{ Hz}}{\text{Hz}}$

(e) Can 50-MeV alpha particles incident upon gold nuclei (Z = 79) penetrate significantly the nuclear charge? The approximate radius of the Au nucleus is about $6 \times 10^{-15}$ m

This is Rutherford Scattering, on our equation sheet we see:

$\Gamma_{\text{min}} = \frac{Z_1 Z_2 e^2}{4 \pi \epsilon_0 K} = \frac{Z_1 Z_2 k e^2}{K} = \frac{2 (\alpha)(1.44 \text{ MeV} \cdot 10^{-15} \text{ m})}{50 \text{ MeV}} = 4.55 \times 10^{-15} \text{ m}$

Since $4.55 \times 10^{-15} \text{ m} < 6 \times 10^{-15} \text{ m}$, the alpha particle just penetrates the nucleus.
(f) A monoenergetic beam of electrons produces a bright reflection spot from the face of a NaCl crystal with an incidence angle relative to the surface of $\theta = 20^\circ$. The spacing of the relevant planes is 0.28 nm. What is the accelerating voltage of the gun that produced these electrons?

\[ e^- \quad \text{Intensity} \quad \text{spot} \]

\[ \text{NaCl} \]

\[ 20^\circ \]

\[ \text{Start w/ conservation of energy:} \]

\[ \text{Total } E_i = \text{Total } E_f \implies E_{PEi} + KE_i = E_{PEf} + KE_f \]

\[ \text{(where } E_{PE} \text{ is electron potential energy)} \]

We know $E_{PEi} = KE_i = 0$, so $E_{PEi} = KE_f$, thus we see $e \cdot q \cdot V = \frac{1}{2}mv^2$.

Setting $p = mv$, $V = \frac{1}{2mq}p^2$.

Using the Bragg condition ($\lambda n = 2ds\sin \theta$) and deBroglie relationship ($\lambda = \frac{h}{p}$), we have $p = \frac{nh}{2ds\sin \theta}$.

Substituting this into $V$:

\[ V = \frac{1}{2mq} \left( \frac{nh}{2ds\sin \theta} \right)^2 \]

\[ \text{---End of Exam---} \]

\[ V = \frac{1}{2(9.1 \cdot 10^{-31} \text{ kg})(1.6 \cdot 10^{-19} \text{ C})} \left( \frac{1 \cdot 6.63 \times 10^{-34} \text{ J s}}{2(1.28 \times 10^{-9} \text{ m}) (\sin 20^\circ)} \right)^2 \approx 41 \text{ Volts} \]