These problems are based on Ch. 1-3 of the text, *Data Reduction and Error Analysis for the Physical Sciences, 2nd. ed.* by Bevington and Robinson.

1. The table below shows results from 25 rolls of a pair of dice.

<table>
<thead>
<tr>
<th>i</th>
<th>x_1</th>
<th>i</th>
<th>x_2</th>
<th>i</th>
<th>x_3</th>
<th>i</th>
<th>x_4</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>11</td>
<td>6</td>
<td>7</td>
<td>11</td>
<td>9</td>
<td>16</td>
<td>12</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
<td>7</td>
<td>12</td>
<td>12</td>
<td>7</td>
<td>17</td>
<td>10</td>
</tr>
<tr>
<td>3</td>
<td>7</td>
<td>8</td>
<td>5</td>
<td>13</td>
<td>5</td>
<td>18</td>
<td>5</td>
</tr>
<tr>
<td>4</td>
<td>6</td>
<td>9</td>
<td>4</td>
<td>14</td>
<td>7</td>
<td>19</td>
<td>8</td>
</tr>
<tr>
<td>5</td>
<td>9</td>
<td>10</td>
<td>8</td>
<td>15</td>
<td>11</td>
<td>20</td>
<td>6</td>
</tr>
</tbody>
</table>

(a) Find the mean, median and most probable values for \( x \) for this distribution.

(b) Find the standard deviation. Note that the mean has been calculated from the data in this case. How did that affect the formula you used to calculate the standard deviation?

2. The probability distribution for the sum of spots showing for a pair of honest dice is:

\[
P(x) = \begin{cases} 
\frac{x - 1}{36} & 2 \leq x \leq 7 \\
\frac{13 - x}{36} & 7 \leq x \leq 12 
\end{cases}
\]

(a) Prove this result.

(b) Find the mean and standard deviation of the distribution (note that the mean is known in this case).

(c) Make a histogram of the distribution in Prob. 1 and indicate the expected values from \( P(x) \) (remember to multiply \( P \) by \( N \) since \( P \) is normalized to 1).

3. We can use the \( \chi^2 \) (Chi-Square) test to estimate the likelihood that the binned data from Prob. 1 come from the distribution of Prob. 2 (i.e., that the dice are honest). Define

\[
\chi^2 = \sum_{i=1}^{11} \frac{(N_i - n_i)^2}{n_i},
\]

\( N_i \) is the contents of the \( i \)th bin and \( n_i \) is the expected value in that bin (need not be an integer). Bin 1 corresponds to \( x = 2 \). A “large” value of \( \chi^2 \) corresponds to a low likelihood. Define the number of *degrees of freedom*, \( \nu \), as the number of sample frequencies (i.e., bins
in the histogram in this case) minus the number of parameters (“constraints”) which have been calculated from the data to describe the probability distribution:

\[ \nu = n_{\text{bins}} - n_c. \]

The expected value of \( \chi^2 \simeq \nu \), or

\[ <\chi^2/\nu> \simeq 1. \]

In this case, \( n_c = 0 \) since we know how many times we rolled the dice and we assumed they were honest; there are no calculated (fitted) parameters in this theoretical distribution.

Find \( <\chi^2/\nu> \) for the histogram in Prob. 2. Is this consistent with honest dice?

4. Evaluate the binomial distribution \( P_B(x; n, p) \) for \( n = 6, p = \frac{1}{6} \) and \( x = 0 \) to 6. Sketch the distribution and identify the mean and standard deviation.

5. A Geiger counter produces an electrical pulse when a charged particle passes through it. The counter has a 200 \( \mu s \) dead time following each pulse. If another particle enters during this time, it will not be counted. \( \beta^- \) particles from a \( ^{137}Cs \) source enter the counter with a constant mean rate of \( 10^3 \) particles per second. The particles arrive randomly in time.

(a) Calculate the mean number of particles, \( \pi_p \), passing through the detector during a random 200 \( \mu s \) interval.

(b) Use Poisson statistics to find the probability that at least one particle (i.e., one or more particles) passes through the detector during a random 200 \( \mu s \) interval. (Hint: relate this to the probability of getting 0 particles.)

(c) Explain why this equals the mean number of counts, \( \pi_c \) in the interval. (Hint: use the definition of the mean (expectation value) of a function, realizing that the number of counts, \( n_c \), will always be 1, no matter how many particles actually pass through during the 200 \( \mu s \) interval).

(d) Now find the efficiency of the counter, defined as the ratio of the mean number of counts to the mean number of particles passing through the counter in a random 200 \( \mu s \) interval.

(e) Repeat the calculation for rates of 2000, 4000, 6000, 8000 and 10000 \( \beta^- \)/s entering the counter and plot a graph of Geiger counter efficiency vs. \( \beta^- \) rate.