1. The figure above shows two ways of connecting a voltage source to an amplifier. The signal connection must travel a distance of \( \approx 30 \text{ cm} \) through a region containing varying electric fields and varying magnetic fields, much as you would find in this room. Small varying voltage differences may also exist between the ground connections. The circuit in (a) uses a single-ended non-inverting amplifier, whereas (b) has a differential-input instrumentation amplifier with common mode gain \( G_{cm} \approx 0 \).

(a) Give examples of three different ways interfering signals could be injected into the input and appear in the output with circuit (a).

(b) Explain why circuit (b) is much less sensitive to each source of interference.

(c) How could circuit (b) be improved further?

(a) See Figure notes i) \( V_c \), ii) \( \Sigma E \), iii) \( \Delta V_a \).

(b) i) Since each wire in the twisted pair sees essentially the same \( E \), \( V_c \) should be the same on each, producing a common mode input on the IA and \( G_{cm} \approx 0 \).

ii) \( \Phi_B \) is minimized by twisting the wires - \( A \) enclosed is small and adjacent twists have opposite orientations, cancelling \( \Phi_B \).

iii) \( \Delta V_a \) appears as a common mode input in (b), unlike (a) in which it appears in series with \( V_+ \) alone (rel. to \( V_- \)).

(c) Provide a shield for the twisted pair. Ground shield at \( V_s \) end. This further reduces \( V_c \).
2. In the midterm exam you were asked to make a simple VI which produced an output sequence \( y_i \) which averaged consecutive samples of an input sampled waveform \( u_i \):

\[
y_i = \frac{1}{2}(u_i + u_{i-1}).
\]

This is in fact a digital filter. Use the techniques described in class (with the convention that the sample period = 1) to do the following:

(a) Find the transfer function \( H(\omega) \) in terms of complex exponentials.

(b) Find an expression for the absolute value \( H(\omega) \equiv |H(\omega)| \equiv \sqrt{HH^*} \) in terms of sinusoids.

(c) Find the value of \( H \) at the Nyquist critical frequency. \( \equiv f_c \)

\[
\begin{align*}
(a) & \quad y_i = \frac{1}{2} \left[ u_i + u_{i-1} \right] = \sum_{k=-N}^{N} C_k u_{i-k} \Rightarrow C_0 = \frac{1}{2}, \quad C_1 = \frac{1}{2} \\
& \quad H(\omega) = \sum_{k=-N}^{N} C_k Z^{-k} \quad \text{where } Z = e^{i\omega} \\
& \quad = \frac{1}{2} Z^0 + \frac{1}{2} Z^{-1} \\
& \quad = \frac{1}{2} e^{i\omega} + \frac{1}{2} e^{-i\omega} = \frac{1}{2} (1 + e^{-i\omega})
\end{align*}
\]

\[
\begin{align*}
(b) & \quad H(\omega) = \sqrt{\left( \frac{1}{2} (1 + e^{-i\omega}) \right)^2 (1 + e^{i\omega})} \\
& \quad = \frac{1}{2} \left[ 1 + e^{i\omega} + e^{-i\omega} + 1 \right]^{1/2} \\
& \quad = \frac{1}{2} \left[ 2 + 2 \frac{e^{i\omega} + e^{-i\omega}}{2} \right]^{1/2} \\
& \quad = \frac{\sqrt{2}}{2} \left[ 1 + \cos \omega \right]^{1/2} = \cos \frac{\omega}{2}. \quad \left( \frac{1 + \cos \theta = \cos^2 \frac{\theta}{2} }{2} \right)
\end{align*}
\]

(c) Since by definition, \( T = 1, \quad f_s = 1, \quad f_c = \frac{f_s}{2} = \frac{1}{2} \)

\( \omega_c = 2\pi f_c = \pi \).

\[
H(\omega) = \cos \frac{\omega}{2} = 0. \quad H(\omega)
\]

\[
\begin{align*}
& \quad 0 \quad \pi \\
& \quad \omega
\end{align*}
\]