Physics 116C Spring 2006
Problem Set 3
Solutions

1. The output of a circuit is grounded locally and connected to a data acquisition system using a 6 ft coaxial cable. The center conductor and shield are connected to instrumentation amplifier differential inputs as shown below.

   (a) Should the shield also connect to the instrumentation amplifier ground at the IA end? No.
   (b) Explain why or why not. Grounding the -IA input forms a complete circuit (ground loop) as shown. If the two ground connections are not at the same potential (due to currents in the ground connection or induced e.m.f. in the loop) a ΔV will appear in series with V_{source}.

2. \[ y_n = \frac{1}{3} u_{n+1} + \frac{1}{3} u_n + \frac{1}{3} u_{n-1} \]

   (a) Find transfer \( H(\omega) \):
   \[
   y_n = \sum_{k=-N}^{N} c_k u_{n-k} = c_{-1} u_{n+1} + c_0 u_n + c_1 u_{n-1}
   \]
   \[ u_n = 1, -1 \leq k \leq 1 \]
   \[ c_{-1} = \frac{1}{3}, c_0 = \frac{1}{3}, c_1 = \frac{1}{3} \]
   \[
   H(\omega) = \sum_{k=-N}^{N} c_k e^{-ik}\omega = \frac{1}{3} e^{i\omega} + \frac{1}{3} + \frac{1}{3} e^{-i\omega} = \frac{1}{3} + \frac{2}{3} \left[ \frac{e^{i\omega} + e^{-i\omega}}{2} \right] = \frac{1}{3} + \frac{2}{3} \cos \omega = \frac{1}{3} [1 + 2 \cos \omega] \]

3. A long 75 Ω transmission line is connected to a pulse generator and is terminated at the far end by a 50 Ω resistor. The voltage on the cable is initially zero. A wave in the form of a 1 V boxcar function 20 ns wide is input and reaches the improperly terminated end.

   (a) Find the amplitude and polarity of the reflected wave.\)
   \[
   V_L = \frac{R_T - Z_0}{R_T + Z_0} V_R = \frac{50\Omega - 75\Omega}{50\Omega + 75\Omega} 1V
   \]

   (b) Find the voltage across the 50 Ω resistor during the pulse.
   \[
   V_L = \frac{-25}{125} V = -0.2 V
   \]

   \[ V_T = V_R + V_L = 1.0 - 0.2 = 0.8 V \]
(a) i) \( \frac{V_r}{i_r} = \frac{2}{50 \, \Omega} = 0.04 \, \Omega \)

ii) For \( 0 < t < 20 \, \text{ns} \),

\[ V_3 = 2 \, \text{V} \]
\[ V_5 = V_K + V_r \quad \text{(KVL)} \]
\[ i_5 = i_r \quad \text{(KCL or q)} \]
\[ V_5 = i_5 R_5 + i_5 Z_0 \]

\[ i_5 = \frac{V_5}{R_5 + Z_0} = \frac{2 \, \text{V}}{100 \, \Omega} = 0.02 \, \text{mA} \]
\[ i_r = 0.02 \, \text{mA} \]
\[ V_r = Z_0 i_r = 50 \, \Omega \times 0.02 \, \text{mA} = 1.0 \, \text{V} \]

(iii) 

(b) First find reflected wave at point c

\[ V_L = \frac{R_T - Z_0}{R_T + Z_0} \]
\[ V_L = -V_r \quad \text{(Shapiro so } R_T = 0) \]
\[ V_L = -1 \, \text{V} \]

Now find right-going and left-going waves at point b, then add them:

Right-going wave

\[ V_r \]

\[ 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7 \quad 8 \quad 9 \quad t (\text{ns}) \]

Left-going wave

\[ V_L \]

Signal = sum
5. (a) \[ H(s) = \frac{1}{s} \]
\[ H(z) = \frac{z+1}{z(z-1)} = \frac{1 + \frac{1}{z}}{1 - \frac{1}{z}} = \frac{\frac{1}{2}z^0 + \frac{1}{2}z^{-1}}{1 - z^{-1}} \]

(b), (c) \[ \frac{c_0 z^0 + c_1 z^{-1}}{1 - d_1 z^{-1}} \Rightarrow c_0 = \frac{1}{2}, c_1 = \frac{1}{2}, d_1 = 1. \]

(d) \[ y_n = \frac{1}{2} u_n + \frac{1}{2} u_{n-1} + y_{n-1} \]

Each value is the sum of the previous value plus \( u_n + u_{n-1} \). This performs an integration using the trapezoidal rule. So this is a reasonable result for an integrator network.