Bevington 2.9:

Bivariate normal distribution, keep track of failures (depressing...)

\[ P = \text{probability of failing} = 0.073 \]

\[ Q = \text{probability of passing} = 1 - P = 0.927 \]

\( n \) = 32

\( P(5 \text{ or more fail}) = 1 - \sum_{x=0}^{4} P_B(x; n, P) \)

\[ P_B(x; n, P) = \frac{n!}{x!(n-x)!} P^x (1-P)^{n-x} \]

\[ P_B(x; 32, 0.073) = \frac{32!}{x!32!} (0.073)^x (0.927)^{32-x} \]

(i.e., prob. everyone passes)

\[ = 0.0884 \]

\[ P_B(1; n, P) = 32 (0.073)(0.927)^{31} = 0.2228 \]

\[ P_B(2; n, P) = \frac{32 \cdot 31}{2} (0.073)^2 (0.927)^{30} = 0.2720 \]

\[ P_B(3; n, P) = \frac{32 \cdot 31 \cdot 30}{6} (0.073)^3 (0.927)^{29} = 0.2142 \]

\[ P_B(4; n, P) = \frac{32 \cdot 31 \cdot 30 \cdot 29}{24} (0.073)^4 (0.927)^{28} = 0.1223 \]

\[ \sum_{x=0}^{4} P_B(x) = 0.9197 \Rightarrow P(x \geq 5) = 1 - 0.9197 = 0.0803 \]

5 or more students would fail 8% of the time.

Bevington 3.1 (bdje)(assume \( u \) and \( v \) are uncorrelated).

3.1 (b):

\[ x = \frac{1}{2} (u - v) \Rightarrow \sigma_x^2 = \sigma_u^2 + \sigma_v^2 \]

where \( a = \frac{1}{2}, b = -\frac{1}{2} \)

\[ = \frac{1}{4} (\sigma_u^2 + \sigma_v^2) \Rightarrow \sigma_x = \frac{1}{2} \sqrt{\sigma_u^2 + \sigma_v^2} \]

\( x = uv^2 \)

\[ \sigma_x^2 = \sigma_u^2 \left( \frac{\partial x}{\partial u} \right)^2 + \sigma_v^2 \left( \frac{\partial x}{\partial v} \right)^2 = \sigma_u^2 (u^2) + \sigma_v^2 (2uv)^2 \]

\[ = u^4 \sigma_u^2 + 4u^2v^2 \sigma_v^2 \Rightarrow \sigma_x = \sqrt{u^2 \sigma_u^2 + 4u^2 \sigma_v^2} \]

3.1 (c):

\[ x = u^2 + v^2 \]

\[ \sigma_x^2 = \sigma_u^2 (2u)^2 + \sigma_v^2 (2v)^2 \Rightarrow \sigma_x = \sqrt{4u^2 \sigma_u^2 + 4v^2 \sigma_v^2} \]
2.13) We know an average of $\mu = 2$ neutrinos are detected each day. We can surmise from this low average number of counts that we should use Poisson statistics.

a) The probability of detecting 8 or more neutrinos in one day would then be:

$$P(x \geq 8) = \sum_{x=8}^{\infty} \frac{\mu^x}{x!} e^{-\mu} = \sum_{x=8}^{\infty} \frac{2^x}{x!} e^{-2} \quad (\text{Equation 1})$$

But since $\sum_{x=0}^{\infty} \frac{\mu^x}{x!} e^{-\mu} = 1$, then:

$$P(x \geq 8) = 1 - \sum_{x=0}^{7} \frac{2^x}{x!} e^{-2} \quad (\text{Equation 2})$$

$$= 1 - e^{-2} \sum_{x=0}^{7} \frac{2^x}{x!}$$

$$= 1 - e^{-2} \times (1 + \frac{2}{1} + \frac{4}{2} + \frac{8}{6} + \frac{16}{24} + \frac{32}{120} + \frac{64}{720} + \frac{128}{5040}) \approx 0.001097 \approx 0.0011$$

So there is a 0.11% chance of detecting 8 or more neutrinos in a day.

b) In a ten minute interval $\mu_{10\text{min}} = \frac{2}{24 + 6} = \frac{1}{72} \approx 0.013889$

As you suspect, the probability of detecting very many neutrinos in a ten minute period is incredibly small so carrying out Equation 2 for $\mu_{10\text{min}} = \frac{1}{72}$ would be quite hard (computers and calculators don’t do well with adding or subtracting incredibly small numbers from relatively large numbers like 1). Let’s use Equation 1, but just calculate the first two terms and see if we have to go any further.

$$P(x=8) = \frac{(1/72)^8}{8!} e^{-\frac{1}{72}} \approx 3.386 \times 10^{-20}$$

$$P(x=9) = \frac{(1/72)^9}{9!} e^{-\frac{1}{72}} \approx 5.227 \times 10^{-23}$$

Since the series of equation 1 converges so fast, we can say

$$P(x \geq 8) \approx 3.4 \times 10^{-20}$$
Problem 1. This VI also reads the student scores in from a test file.
Special problem 2. This problem also calculates the actual mean and standard deviation of the distribution (although one can prove mean = np and variance = npq).
Special problem 3: calculates the probability that $x$ is within the interval from $a$ to $b$; $x$ has normal distribution with mean $= \mu$ and standard deviation $= \sigma$. 

Find 
\[ z = \frac{x - \mu}{\sigma} \]

Cumulative unit normal distribution function (i.e., mean $= 0$, std dev. $= 1$)