1. A student collected data on $y$ as a function of $x$ with standard deviation $\sigma$ (assumed Gaussian). The data are given in Table 1 and plotted in Fig. 1.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
<th>$\sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>68</td>
<td>4</td>
</tr>
<tr>
<td>100</td>
<td>83</td>
<td>4</td>
</tr>
<tr>
<td>200</td>
<td>91</td>
<td>3</td>
</tr>
<tr>
<td>300</td>
<td>96</td>
<td>3</td>
</tr>
<tr>
<td>400</td>
<td>113</td>
<td>4</td>
</tr>
<tr>
<td>500</td>
<td>129</td>
<td>3</td>
</tr>
<tr>
<td>600</td>
<td>144</td>
<td>4</td>
</tr>
</tbody>
</table>

The data were fitted with the function $y = a_0 + a_1 x$. The results of the LabVIEW Levenberg-Marquardt fit are shown below:

(a) Write the results for $a_0$ and $a_1$, each in the form $xxx \pm yyy$, where $xxx$ represents the value (to a reasonable number of significant figures) and $yyy$ represents the standard deviation.

(b) The point at $x_i = 300$ appears to miss the fitted line by $2.29\sigma_i$. What is the probability of a deviation this large or larger (in either direction)? Hint: Remember Table C.2 in Bevington.

(c) How many degrees of freedom ($\nu$) are there for this fit?

(d) Find $\chi^2/\nu$ for the fit. What is the expectation value for this quantity?

(e) Use Table C.4 in Bevington to estimate the probability of exceeding this value of $\chi^2/\nu$.

(f) How can you tell that the errors in $a_0$ and $a_1$ are correlated? What is the value of $\sigma_{01}^2$?

(g) We calculate the area under the curve from $x = 0$ to $x = 600$ using the formula $A = a_0 \Delta x + \frac{1}{2} a_1 (\Delta x)^2$ where $\Delta x = 600$. Find $\sigma_A$. 


2. The $\chi^2$ distribution with $\nu$ degrees of freedom has mean $\nu$ and variance $2\nu$. Make a LabVIEW program to generate samples from a $\chi^2$ distribution with $\nu$ degrees of freedom. Use the transformation method to generate the samples using the uniform distribution random number generator VI and the inverse $\chi^2$ distribution VI. Set the value of $\nu$ and the number of samples desired with front-panel controls. Make a histogram of the resulting samples and investigate the shape of the distribution. Have your VI calculate the mean and standard deviation of the distribution and compare with the expected values for several values of $\nu$. Does the distribution appear to be approximately Gaussian for large values of $\nu$?

3. An Amplitude Modulation (AM) transmitter sending an audio tone of frequency $f_m$ at the maximum amplitude produces an output waveform

$$h(t) = A\cos 2\pi f_c t + A\cos 2\pi f_c t \cos 2\pi f_m t$$

where $A$ is the amplitude of the unmodulated carrier wave of the transmitter and $f_c$ is the carrier wave frequency. We can write $h(t) = h_1(t) + h_2(t)$ where

$$h_1(t) = A\cos 2\pi f_c t \quad \text{and} \quad h_2(t) = A\cos 2\pi f_c t \cos 2\pi f_m t.$$  

We want to find the frequency spectrum. Since the Fourier transform is linear, we can transform each term separately and add the resulting terms in the frequency domain.

(a) Find an expression for $H_2(f)$ in terms of an integral and evaluate the integral if possible.

(b) Let $f_c = 1$ MHz and $f_m = 1$ kHz. At what positive frequencies are $\delta$ functions located due to $H_2(f)$?

(c) In the U.S., AM carrier frequencies are spaced by 10 kHz. What problem can arise if the bandwidth of the modulation signal is not limited to less than 5 kHz?

4. A cosine function $h_1(t) = A\cos(2\pi f_0 t)$ is multiplied by $h_2(t)$, a unit boxcar function (height = 1) extending from $-T_1$ to $T_1$ (assume $T_1 >> 1/f_0$) to give $h_3(t) = h_1(t)h_2(t)$. (Refer to the Table of Fourier Transforms for the mathematical definition of the boxcar function. Note that $f_0$ and $T_1$ are constants.)

(a) Sketch the resulting $h_3(t)$.

(b) Write down the corresponding Fourier transforms, $H_1(f)$ and $H_2(f)$.

(c) Use the convolution theorem to find a mathematical expression for the Fourier transform, $H_3(f)$. *Hint: recall that $\int_{-\infty}^{\infty} f(y)\delta(x-y)dy = f(x)$.*

(d) Sketch the real part of the Fourier transform of $H_3(f)$.

(e) A sinusoidal waveform is sampled at a frequency $\simeq 10$ times the frequency of the waveform and the FFT is found for a sequence of 1024 samples. This is used to make a plot of the one-sided spectral density. What is the significance of the result in Part (d) in interpreting the resulting frequency spectrum? Would you expect to find a single peak in the frequency spectrum in general? Explain briefly.
## Fourier Transforms

### Time Domain

**Transform**

\[ h(t) \]

**Inverse Transform**

\[ h(t) = \int_{-\infty}^{\infty} H(f)e^{2\pi ft} \, df \]

**Linearity**

\[ h(t) = K_1h_1(t) + K_2h_2(t) \quad \Rightarrow \quad H(f) = K_1H_1(f) + K_2H_2(f) \]

**Convolution Theorem**

\[ g(t) \ast h(t) \equiv \int_{-\infty}^{\infty} g(t')h(t-t') \, dt' \]

\[ g(t)h(t) \quad \Rightarrow \quad G(f)H(f) \]

\[ g(t) \ast h(t) \equiv \int_{-\infty}^{\infty} G(f')H(f-f') \, df' \]

### Frequency Domain

**Boxcar in t gives sinc in f**

\[ h(t) = \begin{cases} A & -T_0 < t < T_0 \\ \frac{\alpha}{2} & t = -T_0, T_0 \\ 0 & t < -T_0, t > T_0 \end{cases} \]

\[ H(f) = 2AT_0 \text{sinc}(2\pi T_0 f) \]

**Sinc in t gives boxcar in f**

\[ h(t) = 2Af_0 \text{sinc}(2\pi f_0 t) \]

\[ H(f) = \begin{cases} A & -f_0 < f < f_0 \\ \frac{\alpha}{2} & f = -f_0, f_0 \\ 0 & f < -f_0, f > f_0 \end{cases} \]

**Constant in t gives δ fn in f**

\[ h(t) = K \]

\[ H(f) = K\delta(f) \]

**δ fn in t gives constant in f**

\[ h(t) = K\delta(t) \]

\[ H(f) = K \]

**Sequence of δ fcns in t gives sequence of δ fcns in f**

\[ h(t) = \sum_{n=-\infty}^{\infty} \delta(t-nT) \]

\[ H(f) = \sum_{n=-\infty}^{\infty} \frac{1}{T}\delta(t-\frac{n}{T}) \]

**Cosine in t gives pair of real δ fcns in f**

\[ h(t) = A \cos(2\pi f_0 t) \]

\[ H(f) = \frac{\alpha}{2}\delta(f-f_0) + \frac{\alpha}{2}\delta(f+f_0) \]

**Sine in t gives pair of imaginary δ fcns in f**

\[ h(t) = A \sin(2\pi f_0 t) \]

\[ H(f) = -i\frac{\alpha}{2}\delta(f-f_0) + i\frac{\alpha}{2}\delta(f+f_0) \]

**Gaussian in t gives Gaussian in f**

\[ h(t) = \exp(-at^2) \]

\[ H(f) = \sqrt{\frac{\alpha}{a}} \exp(-\pi f^2/a) \]