LRC Circuits and Resonance

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Brief Outline
Series LRC Circuit and Resonance

- Current magnitude is maximum when reactance $X = 0$

- Circuit with at least one capacitor and one inductor is in resonance when the imaginary part of its impedance (or admittance) equals 0

- Circuit above in resonance when $\omega L - \frac{1}{\omega C} = 0$

\[
\omega_R = \sqrt{\frac{1}{LC}}, \quad f_R = \frac{1}{2\pi} \sqrt{\frac{1}{LC}}
\]

\[
\begin{align*}
V_{\text{out}} &= iR \\
Z &= j\omega L + \frac{1}{j\omega C} + R \\
i &= \frac{V_{\text{in}}}{Z} = \frac{V_{\text{in}}}{R + j[\omega L - \frac{1}{\omega C}]} = \frac{V_{\text{in}}}{R + jX}
\end{align*}
\]
Q and Bandwidth

- Refer to Q definition in text
- The Q derivation for a series LRC circuit was done in class on Friday
- Consider transfer function for series LRC network with output taken across the resistor
  - Gives band pass filter
Bandwidth for Series LRC Band Pass Filter

- Find half-power points $\omega_1, \omega_2$ for band pass filter
- Write $H(j\omega)$ in terms of $\omega_R$ and

$$Q = \frac{1}{R} \sqrt{\frac{L}{C}} = \frac{\omega_R L}{R} = \frac{1}{\omega_R R C}$$
- Half power when $|H(j\omega)| = 1/\sqrt{2}$

$$H(j\omega) \equiv \frac{V_{out}}{V_{in}} = \frac{R}{R + j[\omega L - \frac{1}{\omega C}]} = \frac{1}{1 + jQ[\frac{\omega}{\omega_R} - \frac{\omega_R}{\omega}]}$$

Half power when $Q \left[ \frac{\omega}{\omega_R} - \frac{\omega_R}{\omega} \right] = \pm 1$

$$\omega^2 \pm \frac{\omega_R}{Q} \omega - \omega_R^2 = 0$$

$$\omega = \pm \frac{\omega_R}{2Q} \pm \omega_R \sqrt{\left( \frac{1}{2Q} \right)^2 + 1}$$

Solutions ($\omega > 0$): $\omega_1 = -\omega_R \frac{2Q}{Q} + \omega_R \sqrt{\left( \frac{1}{2Q} \right)^2 + 1}$

$$\omega_2 = \frac{\omega_R}{2Q} \pm \omega_R \sqrt{\left( \frac{1}{2Q} \right)^2 + 1}$$

Bandwidth $\equiv \omega_2 - \omega_1 = \frac{\omega_R}{Q}$
Resonance for Parallel LRC Circuit

- See text: parallel R, L, C driven by current source \( i(t) \)
  
  \[
  V_{\text{out}} = \frac{i}{Y}
  \]

- At resonance, \( Y \) has imaginary part = 0
  
  \[
  Y = \frac{1}{j\omega L} + j\omega C + \frac{1}{R} = \frac{1}{R} \quad \text{at resonance}
  \]

- Again, \( \omega_R = \sqrt{\frac{1}{LC}} \), \( f_R = \frac{1}{2\pi} \sqrt{\frac{1}{LC}} \)

- \( Q \) defined as before now gives (for parallel resonance)
  
  \[
  Q = R\sqrt{\frac{C}{L}} = \frac{R}{\omega_R L} = \omega_R RC
  \]

- Also read about high-Q coils. This can be used to model a high-Q coil with series resistance \( R_s \) in parallel with \( C \) as a parallel RLC circuit with a resistance-free \( L \) and effective parallel resistance \( L/R_s C \)