* Clarify signs in phase shift oscillator example

* Amplifier frequency response
  - Series, shunt capacitor corner freq.
  - Small signal models of BJT and FET showing shunt C's which limit HF response
  - Miller's Theorem

* Gain - BW product

* BJT example (9.54)
* FET example (9.57)
Clarify sign in phase shift oscillator:

\[ AB = -1 \]

assumed negative feedback.

Taking this into account, the amplifier gain \( A = \frac{RF}{R} \). (-1 there already in \( -1 \))

For the RC network

\[
B = \frac{V_i}{V_o} = \frac{\omega^2 R^3 C^3}{\omega RC (\omega^2 R^2 C^2 - 5) + j(1 - 6 \omega^2 R^2 C^2)}
\]

\( B \) must be real for \( AB = -1 \) so

\[ 1 - 6 \omega^2 R^2 C^2 = 0 \Rightarrow \omega = \omega_0 = \frac{1}{\sqrt{6} RC} \]

\[
B = \frac{\omega_0^2 R^2 C^2}{(\omega_0^2 R^2 C^2 - 5)} \quad \omega_0^2 R^2 C^2 = \frac{1}{6}
\]

\[
= \frac{V_o}{V_o - 5} = \frac{1}{1 - 30} = -\frac{1}{29}
\]

\( AB = -1 \) \Rightarrow \( \frac{RF}{R} \left( -\frac{1}{29} \right) = -1\)

\[ RF = 29 R \]

\[ B = \frac{V_i}{V_o} \]

*German physicist, 1881-1956.*
Simplified treatment of coupling capacitors at input or output stages of amplifiers. (Both source and load impedances are important... and must be known for an exact answer. If one is much larger than the other, simplifications are possible.)

High pass network usually looks like this: \( R_0 = \) source resistance, \( C \) could be load or output stage or input impedance of amplifier.

\[ v = v_0 e^{j\omega t}. \quad \text{At high frequency, this is a resistive divider:} \quad v' = \frac{R_L}{R_0 + R_L} v. \]

In general, \( v' = v \frac{R_L}{(R_0 + R_L + \frac{1}{j\omega C})} \]

\[ = \left( 1 + \frac{R_L}{R_0 + R_L} \right) \frac{(R_0 + R_L)}{(R_0 + R_L) + \frac{1}{j\omega C}}. \]

Thus this has a -3dB point relative to the value at high frequency where \( |1/j\omega C| = R_0 + R_L \)

or \( f_c = \frac{1}{2\pi (R_0 + R_L)C}. \)
Similarly, for shunt capacitance, we have:

\[ V' = V_0 e^{j\omega t} \]

At low frequency, \( V' = \frac{R_L}{R_0 + R_L} V \).

The Thevenin equiv. of things to the left of the dotted line is:

\[ V_1 = \frac{R_L V}{R_0 + R_L} \]

So, \( f_c = \frac{1}{2\pi R' C} \).

\[ R' = \frac{R_L R_0}{R_L + R_0} \].

If \( R_L \gg R_0 \) (but \( R_0 \neq 0 \)), \( R' \approx R_0 \).

If \( f_c \) we want \( f_c \) to be large, \( R_0 \) must be small. [We could make \( R_L \) small but if \( R_0 \) is still large, \( R_L / (R_0 + R_L) \) is small - signal reduced by voltage divider at input].

* e.g. HF response of amplifier.

Note we want \( f_c \) to be small for emitter (or source) by pass capacitor.
HF BJT model

\[ C_c = 5 \, \text{pF} \quad C_e = 100 \, \text{pF} \]

\[ G_m = \frac{dI_c}{dV_{be}} = \frac{1}{r_e} \]

\[ r_h \approx h_{fe} r_e \]

What about \( C_e \)?

**Miller's Theorem:**

If \( V_2 / V_1 = k \)

(As for input and output nodes of a voltage amplifier)

\[ Z_1 = \frac{Z_1'}{1-k} \quad Z_2 = \frac{Z_2'}{k-1} \]

If \( Z_1' = 1/j \omega C' \) (Capacitor) and \( k = -A \)

Then \( C_1 = (A+1)C' \), \( C_2 = \frac{A+1}{A} C' \).

HF BJT model with Miller's theorem:

This allows us to separate input and output networks.
Miller's Theorem proof:

\[ V_2/V_1 = k \]

\[ I_1 = \frac{V_1 - V_2}{Z'} \]
\[ = \frac{(1-k)V_1}{Z'} \]
\[ \frac{V_1}{I_1} = Z_1 = \frac{Z'}{1-k} \]

\[ I_2 = \frac{V_2 - V_1}{Z'} = \frac{V_z(1 - \frac{1}{k})}{Z'} \]
\[ \frac{V_2}{I_2} = Z_2 = \frac{Z'}{1 - \frac{1}{k}} = \frac{kZ'}{k-1} \]
Input stage after applying Miller's thm (high freq. case)

\[ \text{Equiv.:} \quad R_g = R_s || R_1 || R_2 \parallel R_T \quad f_c = \frac{1}{2\pi R_g C_g} \]

\[ C_g = C_e + (1+\lambda)C_e \]

(Following text notation - \( S \) must stand for "lag" (low pass) network)

Output stage

\[ V_{out} \quad R_L \]

\[ g_m V_{be} \quad \frac{A+1}{A} C_e \]

Norton: \[ R_C = R_L \]

Thevenin: \[ R_T = R_C || R_L \]

Circuit:

\[ \frac{1}{C_e} = \frac{1}{2\pi R_T \lambda (\frac{A+1}{A})} \]
Note that the output circuit displays approx. const. gain-bandwidth product (assuming LF extends to $\infty$)

$$\text{BW} = \frac{g_m}{r_c}$$

$$A = \frac{g_m}{r_c} = gmT \gg 1 \quad (\text{Av} = -A)$$

Gain-bandwidth product $A \times \text{BW} = \frac{g_m}{r_c} \times \frac{1}{2\pi T \left(\frac{\alpha+1}{\alpha}\right) r_c} = \frac{1}{2\pi r_c C_c}$

If the input circuit BW does not limit (i.e., $>>$ this BW) $g_m, \text{BW}$, then we get this constant gain-BW product

Exact same analysis applies to FETs, except the FET model is slightly different.

Again, split $g_{gd}$ into (via Miller)

$$(A+1)g_{gd} \text{ in input circuit}$$

parallel to $g_{gs}$, $\left(\frac{A+1}{A}\right)g_{gd} \text{ in output circuit} \parallel g_{ds}$.
Terminology: \( C_e = C_{bc} \), \( C_i = C_{be} \)

\[ f_T = \frac{1}{2\pi(C_e + C_i)R_2} \]

(a) Lower cutoff frequency: treat each part separately to see if any one part is dominant:

i) Input circuit (assume \( C_i \to \infty \))

\[ f_c = \frac{1}{2\pi(R_s + R_{il}R_{ll})(C_1)} \]

\[ R_{il} = \frac{h_{fe}R}{2200 \Omega} = 2200 \Omega \]

\[ = \frac{1}{2\pi(1k\Omega + 177k\Omega)\times 1nF} = 58 \text{ Hz} \]

ii) Output circuit:

\[ f_c = \frac{1}{2\pi(R_e + R_l)C_e} = \frac{1}{2\pi \times 10k\Omega \times 4.7nF} = 3.4 \text{ Hz} \]

iii) Emitter circuit HF \( f_c \) due to \( C_e \) shunt capacitance, assuming \( C_i \to \infty \) (ineffective below \( f_c \)):

\[ R_{equiv} = \text{resistance looking into emitter} = R_e + \frac{R_{il}R_{ll}R_s}{h_{fe} + 1} \]

\[ = 22 \Omega + \frac{1}{100}(900 \Omega) = 31 \Omega \]

\[ f_c = \frac{1}{2\pi(R_{equiv}R_e)C_e} = \sqrt{113 \text{ kHz}} \text{ (determines } \text{ LF -3dB point)} \]
9.54 (b) Upper cutoff frequency

1. Consider input (base) circuit with Miller effect w.r.t. $C = C_{CB}$. We need to know mid-band gain, $A = -A_{m\text{ }\text{mid-band}} = \frac{R_{\text{IL}}R_{E}}{R_{E}}$ for this circuit

   $A = \frac{2.5\text{ k}\Omega}{22\text{ k}\Omega} = 0.114$

   The text defines quantities which are equivalent to what was done in class:
   
   $C_g \equiv \frac{2\pi f_B}{R_g}$

   Where $R_g = R_{S} || R_{P} \ || R_{T}$ and $R_P = R_{T} \ || R_2$ (assumes $R_E$ bypassed)

   and $C_g = C_e + (1 + A)C_c \quad (\equiv C_{BE} + (1 - A)C_{BC})$

   Now we need $C_e$, but we have

   $C_e = \frac{1}{2\pi f_T \cdot R_e} = \frac{1}{2\pi \times 150 \text{ MHz} \times 22 \Omega} = 48 \text{ pF}$

   $C_e = 48 \text{ pF} - 5 \text{ pF} = 43 \text{ pF}$.

   $C_g = 43 \text{ pF} + 115 \times 5 \text{ pF} = 618 \text{ pF}$

   $f_c = \frac{1}{2\pi 2g_{c}} = \frac{1}{2\pi \times 6.39 \text{ s} \times 6.8 \text{ pF}} = 403 \text{ kHz}$.

ii. Consider output (collector) circuit with Miller effect.

   Again, the text defines quantities which are equivalent to what was done in class:

   $f_T = \frac{2\pi f_B}{R_{T} C_T}$ where

   $R_T = R_{\text{IL}} L$ and $C_T = \frac{1 + A}{A} C_c \quad (\equiv \frac{A_{m\text{ }\text{mid-band}}}{A} C_{BE})$

   $f_c = \frac{1}{2\pi \times 2.5 \text{ k}\Omega \times \frac{115}{114} \times 5 \text{ pF} \times 12.6 \text{ MHz}} = 4.136 \text{ kHz}$

   Thus the input circuit dominates and the

   $HF \ f_c = 403 \text{ kHz}$. 

(c) Bandwidth = 403 kHz - 1 kHz = 402 kHz.
(a) Lower cutoff frequency:

i) Input circuit (assume \(C_s \rightarrow \infty\))

\[
 f_c = \frac{1}{2\pi (R_s + R_{11}R_2)C_1} = \frac{1}{2\pi (1000 + 500k) \times 1\mu F} = 0.3 \text{ Hz}
\]

ii) Output circuit: assume \(C_s \rightarrow \infty\) and take \(R_d\) into account (see p. 614):

\[
 f_c = \frac{1}{2\pi (R_0 || R_d + R_L)C_2} = \frac{1}{2\pi \times 9.29k \times 1\mu F} = 17 \text{ Hz}
\]

iii) Source circuit: HF \(f_c\) due to \(C_3\) shunt, taking \(R_d\) into account (see pp. 615-616):

\[
 f_c = \frac{1}{2\pi R_0 \cos C_s} \left\{ \text{ where } R_0 = R_s || \frac{1}{g_m} || R_d, \right\} = \frac{1}{2\pi \times 769.5 \times 1\mu F} = 207 \text{ Hz}, \quad \text{determines LF -3dB point.}
\]

(b) Upper cutoff frequency: See pp. 621-623. The FET circuit is related to the circuit of the BJT common emitter amplifier (see prob. 9.54 solution):

i) Input

\[
 f_c = \frac{1}{2\pi R_0 C_g} \left\{ R_0 = R_s || R_{11}R_2 \leq 100 \text{ k}\Omega \right. \]

\[
 C_g = C_{gs} + (1 + A) C_{gd}
\]

\[
 A = g_m [R_d || R_{11}R_2]
\]

\[
 g_m = 0.8 \text{ mV} \\
 R_s = 4 \text{ pF} \\
 C_{gd} = 2 \text{ pF} \\
 C_{gs} = 1 \text{ pF} \\
 R_d = 10 \text{ k}\Omega
\]

\[
 A = 0.8 mV 	imes 2.31 k\Omega = 1.85
\]

\[
 C_g = 4 \text{ pF} + 2.85 \times 2 \text{ pF} = 9.7 \text{ pF}
\]
9.15 (b) ii) Output circuit, again related to CE BJT analysis,

\[ f_c = \frac{1}{2\pi R_C} \left[ \frac{1}{R_T} \right] \]

where \( R_T = R_d || R_D || R_L = 2.3 \) kΩ

\[ R_C = C_{ds} + \frac{1 + A}{A} C_{gd} \]

\[ = 1 \text{pF} + \frac{2.85}{1.85} \times 2 \text{pF} \]

\[ = 4.1 \text{pF} \]

\[ f_c = 16.8 \text{MHz} \]

\[ \text{determines } \text{HF cutoff} \]

(c) \( BW = f_{c_{\text{HIGH}}} - f_{c_{\text{LOW}}} = 16.8 \text{MHz} \)