Physics 116A Notes
Fall 2003
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• References:
  – Text for course:
  – Others as noted
Physics 116A, 10/15/03: Outline

- Review and complete Monday lecture

- Phasors

- Circuit relations work for phasor analysis too with \( V = IZ \) as before, e.g.,
  - KVL and KCL
  - Series and parallel impedances
    * Implication for C's in series and parallel since \( Z_C = \frac{1}{j\omega C} \)
  - Nodal and mesh analysis, superposition
  - Thévenin equivalent

- Examples, (e.g., Text, Prob. 4.7, set up 4.18)

- Next: AC power
  - Reactance, rms quantities, power factor, maximum power transfer
Phasors

- **Phasor**: a representation of an AC quantity at a given $\omega$ as a magnitude (or amplitude) $A$ and initial phase angle $\phi$, **without the** $e^{j\omega t}$ **term** (which cancels in an equation involving AC analysis anyway).

- Text uses upper case bold face letters for phasors

- Phasors are complex numbers, usually expressed in polar form, just not functions of $t$
  
  - remember that vector addition (or more properly, complex number addition) is required to add phasors

- You can put the time dependence back in by multiplying by $e^{j\omega t}$ where appropriate (or find the real part as fcn of $t$ as $A\cos(\omega t + \phi)$)

- **Example** (capacitor driven by $v(t) = V_0e^{j\omega t}$):
  
  $$i(t) = \frac{v(t)}{Z_C} = j\omega CV_0e^{j\omega t} = \omega CV_0e^{j\frac{\pi}{2}}e^{j\omega t}$$

- Current expressed as phasor (text uses upper case bold face letters for phasor variables):
  
  $$I = \omega CV_0 \angle 90^\circ$$

- And the actual time-dependent current is $i(t) = \omega CV_0 \cos(\omega t + \frac{\pi}{2})$
The circuit and phasor diagram are shown above

- We say $I$ leads $V = V_0 \angle 0$ by $90^\circ$ here
- If $I$ is considered the output, then this is called a lead network
- Compare with results if $C$ is replaced with $R$ or $L$
  * With $R$, $I$ and $V$ in phase; With $L$, $I$ lags $V$ by $90^\circ$

- Text refers to phasor analysis as working in the frequency domain
  - The analysis assumes sinusoids at a single $\omega$ (or $f \equiv \frac{\omega}{2\pi}$)
  - The only dependence left is on $\omega$ (or $f$)
  - Bode plot shows amplitude and phase response as fcn of $\omega$ (or $f$)
AC Analysis Example

(a) Find $V_0(t)$ using voltage division.

(b) Draw phasor diagram (showing $V_S$ and $V_0$).

- Is this a lag or a lead network?

(a) Voltage divider (with impedances):

$$V_0 = \frac{R_L V_S}{R_L + R_1}$$

$$Z_L = j\omega L = 12 \cdot j \Omega$$

$$R_L Z_L = \frac{R_L Z_L}{R_L + Z_L} = \frac{5 \times 12}{5 + 12} \Omega = \frac{60 \times 90^\circ \cdot \Omega}{13 \times 67.4^\circ}$$

$$= 4.6 \times 22.6^\circ \Omega = 4.6 \cos 22.6^\circ + j 4.6 \sin 22.6^\circ$$

$$= 4.23 + 1.77 j$$

$$V_0 = 4.23 \cdot 22.6^\circ \cdot (13 \times -22.6 \text{ V})$$

$$= 59.8 \text{ V}$$

$$= 6.4 \cos (2t - 0.19) \text{ V} \left( 10.9^\circ = 0.19 \text{ radians} \right)$$
AC Analysis Example (continued)

Prob. 4.7 (b): Phasor diagram

Note: \( \text{ang}(V_o) > \text{ang}(V_s) \) so \( V_o \) leads \( V_s \).

\(-22.6^\circ + 11.7^\circ = \text{ang}(V_o) - \text{ang}(V_s)\)

\( |V_o| = 10 V \)

\( |V_s| = 18 V \)

This is a lead network.
AC Thévenin Example

Example: \( V_{12} \) in Bobrow

(a) Find Thévenin equivalent of stuff inside dotted lines.
(b) Find \( V_{ac} \).

\[
V_s = 4 \cos(2t - 60^\circ) \ V.
\]
\( \omega = 2 \ \text{rad/s}. \)

(c) Find open circuit voltage. In this case no current flows through the 2H inductor, so \( V_{ac} = V_{bc} \) and \( V_{bc} \) can be found with voltage divider formula:

\[
\frac{V_{bc}}{V_{bc} \ \text{open circuit}} = \frac{Z_c}{Z_{ac}} \ \frac{V_s}{Z_c} \quad \text{and} \quad Z_c = j \frac{1}{\omega L} = -j \frac{\frac{1}{2}}{2} = -8j \ \Omega
\]

\[
V_{bc} = \frac{-j}{8 - 8j} V_s = \frac{-j}{1-j} (4 \times -60^\circ) \ V
\]

\[
= (1 + 90^\circ) \times (4 + 60^\circ) \ V
\]

\[
= \left( \frac{\sqrt{2}}{2} \right) (4 - 105^\circ) \ V = (2\sqrt{2} \times -105^\circ) \ V.
\]

\[
V_{ac} = 2\sqrt{2} \times -105^\circ \ V.
\]
AC Thévenin Example (continued)

4.18 (continued)

Now set $V_s = 0$ and find the impedance between $a$ and $b$ inside the dashed lines:

With $V_s = 0$ (replace $V_s$ with short circuit) $R \| C$.

\[ Z_o = Z_L \| Z_C = 4 \Omega + \frac{8 \Omega \cdot (-8j)}{8 - 8j} \]
\[ = 4 \Omega - \frac{8j}{1-j} = \frac{(4j + 4 - 8j)\Omega}{1-j} = 4 \frac{(1-j)}{1-j} \Omega \]

So we have \[ Z_o = 4 \Omega \]

Voltage divider:

\[ V_o = \frac{4\Omega}{4\Omega + 4\Omega} V_{oc} = \frac{1}{2} V_{oc} = \sqrt{2} V \cos(2\pi - 105^\circ) \]

(or $V_o = 0.707 \cos(2\pi - 105^\circ) V$)