1. Gaussian wave functions (30 points)
Consider the wave function
\[ \psi(x) = Ke^{ax-bx^2} \]
where \( K, a, \) and \( b \) are constants.

a. Use the condition of normalization to find \( K \).

b. Find the expectation values \( \langle x \rangle \) and \( \langle x^2 \rangle \).

c. Find the expectation values \( \langle p \rangle \) and \( \langle p^2 \rangle \).

d. Find the standard deviations (“uncertainties”) \( \Delta x \) and \( \Delta p \).

e. Show that the Heisenberg uncertainty relations hold.

f. Find the probability current \( J(x) \).

2. Probabilities (20 points)
Consider a wave function
\[ \psi(x) = \begin{cases} 
0 & x < -1 \\
A(-1 + x^2) & -1 < x < 1 \\
0 & x > 1 
\end{cases} \]

(You might want to sketch \( \psi \) and \( |\psi|^2 \) to help visualize the problem.)

a. Find the constant \( A \) from the normalization requirement.

b. Let \( b \) be an arbitrary number between \(-1 \) and \( 1 \). What is the probability that the particle is located in the range \(-1 < x < b\)?

c. If you answered question b correctly, you should find that the probability is one when \( b = 1 \). Explain why.